

Algebraic Expressions and Integers

Chapter

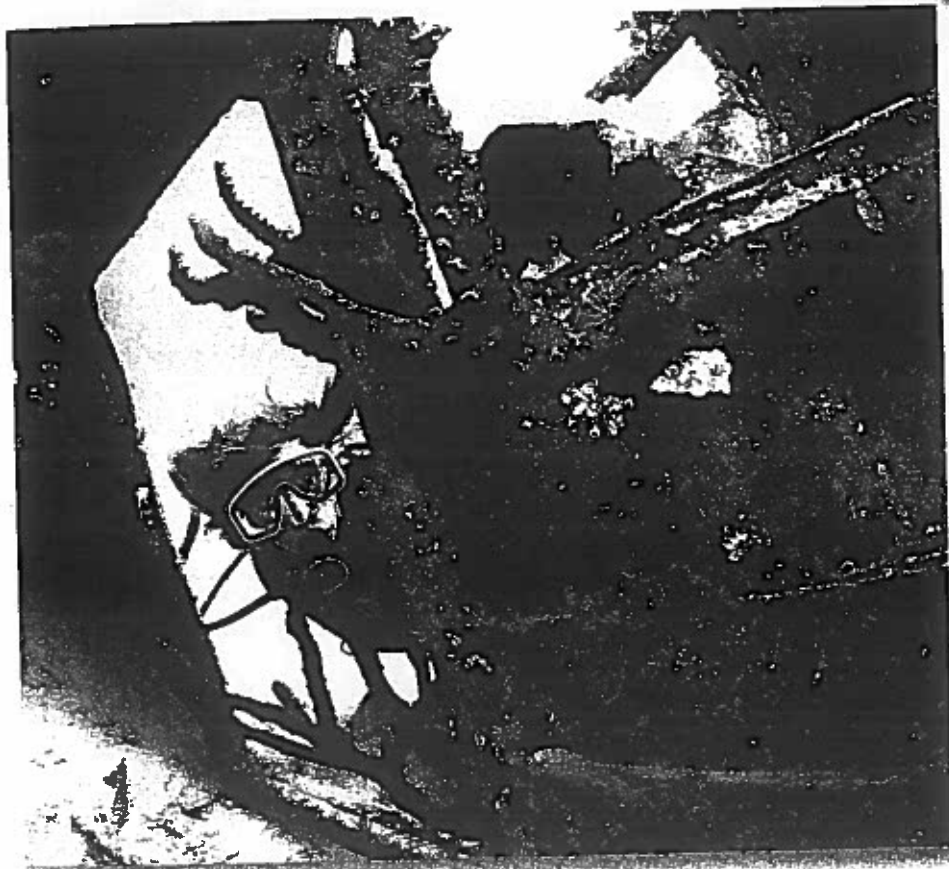
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LESSONS

- 1-1 Variables and Expressions
- 1-2 The Order of Operations
- 1-3 Writing and Evaluating Expressions
- 1-4 Integers and Absolute Value
- 1-5 Adding Integers
- 1-6 Subtracting Integers
- 1-7 Inductive Reasoning
- 1-8 Problem Solving: Look for a Pattern
- 1-9 Multiplying and Dividing Integers
- 1-10 The Coordinate Plane

Key Vocabulary

- absolute value (p. 19)
- conjecture (p. 35)
- coordinate plane (p. 52)
- counterexample (p. 37)
- evaluate (p. 14)
- inductive reasoning (p. 35)
- integers (p. 19)
- opposites (p. 19)
- order of operations (p. 8)
- ordered pair (p. 52)
- origin (p. 52)
- quadrants (p. 52)
- variable (p. 4)
- variable expression (p. 4)
- x -axis (p. 52)
- x -coordinate (p. 52)
- y -axis (p. 52)
- y -coordinate (p. 52)



What You'll Learn Next

In this chapter, you will learn how to

- Use variables and variable expressions.
- Perform operations with integers.
- Graph points in the coordinate plane.
- Solve a problem by looking for a pattern.



Activity Lab Applying what you learn, on pages 64–65 you will solve problems about sunken ships.

1-1

Variables and Expressions

What You'll Learn

- To identify variables, numerical expressions, and variable expressions
- To write variable expressions for word phrases

... And Why

To use the language of algebra to model real-world problems

Check Skills You'll Need

Complete each equation.

- 1 week = ■ days
- 1 foot = ■ inches
- 1 nickel = ■ cents
- 1 gallon = ■ quarts
- 1 yard = ■ feet

GO for Help

Table 1, p. 798

New Vocabulary

- variable
- variable expression

1 Identifying Numerical and Variable Expressions

Gas Mileage How many miles can you drive on ten gallons of gas? The answer depends on the type of vehicle you drive. The table shows some typical data.

Vehicle Type	Miles	Gallons	Miles per Gallon
Subcompact	330	10	$330 \div 10$
Compact	300	10	$300 \div 10$
Mid-size sedan	245	10	$245 \div 10$
Sport utility vehicle	175	10	$175 \div 10$
Pickup truck	160	10	$160 \div 10$

The last column gives a *numerical expression* for each vehicle's miles per gallon.

If you don't know the number of miles, you can use a *variable* to stand for the number. Then you can write a *variable expression* for miles per gallon.

variable $\rightarrow m \leftarrow$ miles on 10 gallons

variable expression $\rightarrow m \div 10 \leftarrow$ miles per gallon

A **variable** is a letter that stands for a number.

A **variable expression** is a mathematical phrase that uses variables, numerals, and operation symbols.

1 EXAMPLE Identifying Expressions

Identify each expression as a *numerical expression* or a *variable expression*. For a variable expression, name the variable.

a. $5 - 5$
numerical expression

b. $c - 5$
Variable expression; c is the variable.

Online
active math



For: Variable Expressions Activity
Use: Interactive Textbook, 1-1

✓ Quick Check

1. Identify each expression as a *numerical expression* or a *variable expression*. For a variable expression, name the variable.

- a. $8 \div x$ b. 100×6 c. $d + 43 - 9$

2 Writing Variable Expressions

You can translate word phrases into variable expressions.

Word Phrase	Variable Expression
Nine more than a number y	$y + 9$
4 less than a number n	$n - 4$
A number z times three	$z \cdot 3$ or $3z$ or $3(z)$
A number a divided by 12	$a \div 12$ or $\frac{a}{12}$
5 times the quantity 4 plus a number c	$5 \cdot (4 + c)$ or $5(4 + c)$

Writing in Math

You can translate many words for operations into operation symbols.

total	+
more than	+
increased by	+
difference	-
fewer than	-
less than	-
decreased by	-
product	\times or \cdot or $()$
times	\times or \cdot or $()$
quotient	\div or $-$
divided by	\div or $-$

A variable expression is an efficient way to express a mathematical relationship.

2 EXAMPLE Real-World Problem Solving

Science The fastest dinosaur may have been *Ornithomimus*, which could run about 60 ft in a second. Write a variable expression for the distance *Ornithomimus* could run in a given time.

Words 60 times number of seconds



Let s = number of seconds.

Expression 60 \cdot s

The variable expression $60 \cdot s$, or $60s$, describes the distance in feet *Ornithomimus* could run in s seconds.



Real-World Connection

Ornithomimus was an ostrich-like oviraptor about 7 ft tall. Its long tail acted as a counterbalance and as a stabilizer during fast turns.

✓ Quick Check

2. a. Bagels cost \$.50 each. Write a variable expression for the cost of b bagels.
 b. **Measurement** Write a variable expression for the number of hours in m minutes.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example



Example 1
(page 4)

Identify each expression as a *numerical expression* or a *variable expression*. For a variable expression, name the variable.

1. $b + 6$

2. $80 \div 8$

3. $14 - n$

4. 14×14

5. $100x$

6. $8 + 8 + 8 + 8$

Example 2
(page 5)

Write a variable expression for each word phrase.

7. 16 more than m

8. 6 divided by z

9. the product of c and 3

10. 2 less than p

11. b times 3

12. 4 fewer than j

13. n divided by 3

14. 3 divided by n

15. x less than 2

16. 8 less than z

Write a numerical or variable expression for each quantity.

17. two dozen eggs

18. d dozen eggs

19. the value in cents of 7 nickels

20. the value in cents of n nickels

21. number of quarts in 3 gallons

22. number of quarts in g gallons

B Apply Your Skills

Identify each expression as a *numerical expression* or a *variable expression*. For a variable expression, name the variable.

23. $d + 53$

24. $12 - 7$

25. $\frac{g}{9}$

26. $4(5)$

Measurement Write an expression for each quantity.

27. the number of days in 4 weeks

28. the number of days in w weeks

29. number of pounds in 160 ounces

30. number of pounds in z ounces

31. the number of feet in 100 inches

32. the number of feet in i inches

33. Mia has \$20 less than Brandi. Brandi has d dollars. Write a variable expression for the amount of money Mia has.

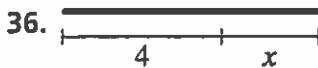
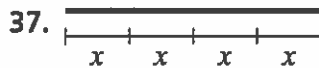
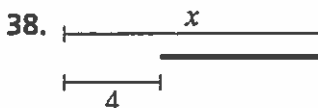
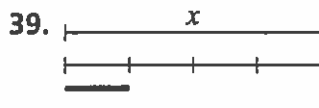
Use the calorie chart at the left for Exercises 34 and 35.

Food	Calories
Bread slice	55
Apple	70
Banana	100
Egg	110

34. Write a variable expression for the number of calories in e eggs and one slice of bread.

35. Write a variable expression for the number of calories in a fruit salad made from a apples and b bananas.

Modeling In each model, the red line represents a variable expression. Match each model with its expression.

- A. $\frac{x}{4}$ B. $4 + x$ C. $4x$ D. $x - 4$
36. 
37. 
38. 
39. 

C Challenge

40. **Writing in Math** How are numerical expressions and variable expressions similar? How are they different?
41. **Error Analysis** A student wrote the variable expression $n - 5$ for the word phrase n less than five. Explain the student's error.



Test Prep

Multiple Choice

A hot-air balloon is at an altitude of m meters. In Exercises 42–44, which expression matches the given word phrase?

42. the balloon's new altitude after rising 34 meters
 A. $m - 34$ B. $m + 34$ C. $3m$ D. $34m$
43. the balloon's new altitude after falling 2,000 meters
 F. $m + 2,000$ G. $2,000 - m$ H. $2,000m$ J. $m - 2,000$
44. the balloon's new altitude after tripling its altitude
 A. $m - 34$ B. $m + 34$ C. $3m$ D. $34m$
45. Pam is 15 years old. Which expression gives Pam's age p years ago?
 F. $p - 15$ G. $p + 15$ H. $15 - p$ J. $\frac{p}{15}$

Mixed Review

Skills Handbook

Compute.



46. 9×25 47. $3 \times 6 \times 4$ 48. 8×1
 49. $225 \div 3$ 50. $169 \div 13$ 51. $25,942 \div 12,971$

52. **Purchasing** A customer buys orange juice for \$.95 and two apples for \$.55 each. She gives the cashier a five-dollar bill. How much change should the cashier give the customer?
53. **Nutrition** There are seven servings in a box of pita chips. The box weighs 16 oz. About how many ounces are in each serving? Round your answer to the nearest tenth.

The Order of Operations

What You'll Learn

- To use the order of operations
- To use grouping symbols

... And Why

To find the value of an expression with more than one operation

Check Skills You'll Need

Find each quotient.

- $164 \div 2$
- $344 \div 8$
- $284 \div 4$
- $133 \div 7$
- $182 \div 13$
- $650 \div 25$

GO for Help

Skills Handbook, p. 782

New Vocabulary

- order of operations

1 Using the Order of Operations

Activity

Experimenting With Order

In most languages, the meaning of words depends on their order. For example, "sign the check" is not the same as "check the sign."

Similarly, order is important in the language of mathematics.

- Mental Math** Find the value of the expression $3 + 5 \times 2$.
- Analyze** What answer do you get to Question 1 if you multiply before adding? If you add before multiplying?
- Reasoning** How does the order in which you do the operations affect your answer?

The order in which you perform operations can affect the value of an expression. To avoid confusion, mathematicians have agreed on an **order of operations**. Multiply and divide first. Then add and subtract.

To *simplify* a numerical expression, you use the order of operations and replace the expression with the simplest name for its value.

1 EXAMPLE Simplifying Expressions

Simplify $4 + 15 \div 3$.

$$4 + 15 \div 3$$

$$4 + 5$$

First divide.

$$9$$

Then add.

Quick Check

- Simplify each expression.

a. $2 + 5 \times 3$

b. $12 \div 3 - 1$

c. $10 - 1 \cdot 7$

When operations have the same rank in the order of operations, do them from left to right.

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2 EXAMPLE Using the Order of Operations

Simplify $3 \cdot 5 - 8 \div 4 + 6$.

$$3 \cdot 5 - 8 \div 4 + 6$$

$$15 - 2 + 6$$

Multiply and divide from left to right.

$$13 + 6$$

Add and subtract from left to right.

$$19$$

Add.

✓ Quick Check

2. Simplify each expression.

- a. $4 - 1 \cdot 2 + 6 \div 3$ b. $5 + 6 \cdot 4 \div 3 - 1$



Calculator Hint

Many calculators use the order of operations. To test yours, enter $10 - 4 \div 2$. If the answer is 8, then your calculator uses the order of operations. If the answer is 3, then your calculator does not use the order of operations.

2 Using Grouping Symbols

Grouping symbols, such as parentheses, (), and brackets, [], indicate order. A fraction bar also is a grouping symbol, since $\frac{4+2}{3} = (4+2) \div 3$. Always work inside grouping symbols first.

Key Concepts Order of Operations

1. Work inside grouping symbols.
2. Multiply and divide in order from left to right.
3. Add and subtract in order from left to right.

3 EXAMPLE Simplifying With Grouping Symbols

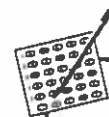
Multiple Choice Which procedure is correct for simplifying $24 \div [6 - (2 \cdot 2)]$?

- (A) $24 \div [6 - (2 \cdot 2)] = 4 - 2 \cdot 2 = 4 - 4 = 0$
(B) $24 \div [6 - (2 \cdot 2)] = 24 \div 4 \cdot 2 = 24 \div 8 = 3$
(C) $24 \div [6 - (2 \cdot 2)] = 4 - 2 \cdot 2 = 2 \cdot 2 = 4$
(D) $24 \div [6 - (2 \cdot 2)] = 24 \div (6 - 4) = 24 \div 2 = 12$

Choice D follows the order of operations by working inside the grouping symbols first. Choice D is correct.

✓ Quick Check

3. Simplify the expression $1 + \frac{10 - 2}{4}$.

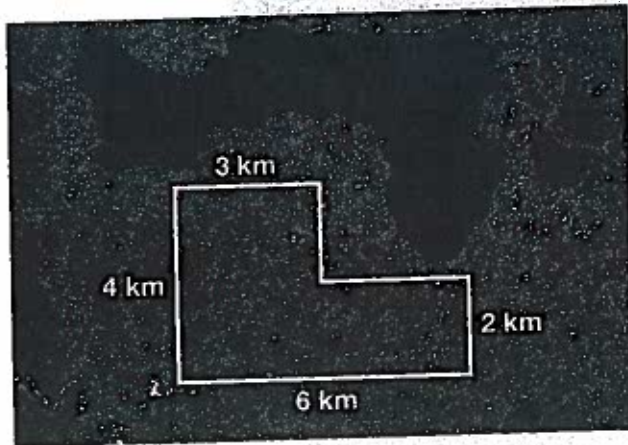


Test-Taking Tip

Grouping symbols say "do this first." Inside grouping symbols, multiply and divide before adding and subtracting.

You can use the order of operations to find the area of an irregular figure by more than one method.

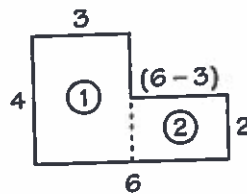
More Than One Way



Urban Planning Some urban planners specialize in planning entire new towns. These towns are designed for livability, with plenty of open space. The sketch shows the dimensions for a new town called Panorama. Find Panorama's area.

Kevin's Method

Divide the figure into rectangles. Then add their areas.



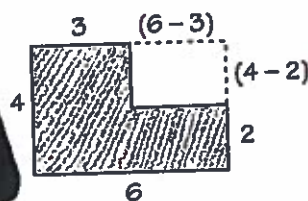
$$\begin{aligned} \text{Area} &= \text{Area } \textcircled{1} + \text{Area } \textcircled{2} \\ &= 4 \cdot 3 + (6 - 3) \cdot 2 \\ &= 4 \cdot 3 + 3 \cdot 2 \\ &= 12 + 6 \\ &= 18 \end{aligned}$$

Panorama's area is 18 km^2 .



Tina's Method

Visualize attaching a small rectangle to complete a large rectangle. Then subtract the small area from the large area.



$$\begin{aligned} \text{Area} &= \text{Area of large rectangle} - \text{Area of small rectangle} \\ &= 6 \cdot 4 - (6 - 3) \cdot (4 - 2) \\ &= 6 \cdot 4 - 3 \cdot 2 \\ &= 24 - 6 \\ &= 18 \end{aligned}$$

Panorama's area is 18 km^2 .



Choose a Method

1. Which method would you use to find the town's area? Explain.
2. Can you think of another way to solve the problem? Explain.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Examples 1 and 2
(pages 8 and 9)



Example 3
(page 9)

Simplify each expression.

- | | | |
|------------------------------|------------------------------|--------------------------------|
| 1. $3 + 6 \times 4$ | 2. $35 \div 7 - 2$ | 3. $8 - 2 \cdot 3$ |
| 4. $6 - 6 \div 3$ | 5. $21 - 13 + 8$ | 6. $6 \cdot 2 + 4$ |
| 7. $12 - 8 \div 2 + 3$ | 8. $21 \div 7 + 14 \times 2$ | 9. $2 \cdot 2 + 0 \cdot 4$ |
| 10. $4(4) - 2(5)$ | 11. $2 + 3 \cdot 24 \div 6$ | 12. $4 \div 4 \cdot 4 + 4 - 4$ |
| 13. $7 + 3 \cdot (8 \div 4)$ | 14. $2(15 - 9) \cdot 9$ | 15. $[2 + (6 \cdot 8)] - 1$ |
| 16. $2(6) + \frac{7+8}{3}$ | 17. $3(7 + 4)$ | 18. $12 \div (3 - 2) + 1$ |
| 19. $6 + \frac{6+2}{4}$ | 20. $\frac{21+15}{3+6}$ | 21. $(21 + 3) \div 4 \div 2$ |

B Apply Your Skills

22. **Error Analysis** A student found the value of the expression $30 \div 6 - 1$ to be 6. Explain the student's error.

23. **Writing in Math** Why do we need to agree on an order of operations?

Simplify each expression. Justify your work.

- | | | |
|--------------------------|-----------------------------|--------------------------|
| 24. $(56 - 5) \div 17$ | 25. $60 \div 4 + 9$ | 26. $2[8 + (5 - 3)] - 8$ |
| 27. $12 \div 3 \times 4$ | 28. $36 - 27 \div 9 \div 1$ | 29. $6(4 + 1) - 5$ |
| 30. $14 + 5 \times 2$ | 31. $440 \div (2 + 18)$ | 32. $16 \div 8 \times 2$ |

Compare. Use $>$, $<$, or $=$ to complete each statement.

- | | |
|---|---|
| 33. $15 \cdot 3 - 2 \square 15 \cdot (3 \div 2)$ | 34. $18 - 6 \div 3 \square (18 - 6) \div 3$ |
| 35. $8 + 12 \div 4 \square (8 + 12) \div 4$ | 36. $22 - 7 \cdot 2 \square (22 - 7) \cdot 2$ |
| 37. $12 \div 3 + 9 \cdot 4 \square 12 \div (3 + 9) \cdot 4$ | |
| 38. $(19 - 15) \div (3 + 1) \square 19 - 15 \div 3 + 1$ | |

Insert grouping symbols to make each number sentence true.

39. $7 + 4 \cdot 6 = 66$ 40. $7 \cdot 8 - 6 + 3 = 17$ 41. $3 + 8 - 2 \cdot 5 = 45$

42. **Multiple Choice** Insert grouping symbols to make the number sentence $3 + 8 - 2 \cdot 5 = 45$ correct.

- | | |
|----------------------------------|----------------------------------|
| (A) $(3 + 8) - 2 \cdot 5 = 45$ | (B) $3 + (8 - 2) \cdot 5 = 45$ |
| (C) $[3 + (8 - 2)] \cdot 5 = 45$ | (D) $3 + [8 - (2 \cdot 5)] = 45$ |

Write a numerical expression for each phrase. Then simplify.

43. five added to the product of four and nine
44. twenty-one minus the sum of fifteen and five
45. seventeen minus the quotient of twenty-five and five

GO for Help

For a guide to solving Exercise 26, see page 13.

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Understanding Math Problems Read the problem below. Then let Daria's thinking guide you through the solution. Check your understanding with the exercises at the bottom of the page.

Simplify $2[8 + (5 - 3)] - 8$.

What Daria Thinks

What Daria Writes

There are two sets of grouping symbols, square brackets and parentheses: $[8 + (5 - 3)]$.

$$2[8 + (5 - 3)] - 8$$

I need to begin by working inside the innermost grouping symbols, the parentheses: $(5 - 3)$.

$$2[8 + (2)] - 8$$

Now I need to work inside the next pair of grouping symbols, the square brackets, to find $[8 + 2]$.

$$2[10] - 8$$

Multiply before subtracting.

$$20 - 8$$

Now subtract.

$$12$$

The simplified expression is 12.

EXERCISES

Simplify each expression.

- | | | |
|-----------------------------------|-----------------------------------|---------------------------|
| 1. $2[(13 - 4) \div 3]$ | 2. $1 + \frac{10 - 2}{4}$ | 3. $3[(8 + 4) \div 6]$ |
| 4. $\frac{6 + 9}{3} - 2$ | 5. $4[3 + (2 \cdot 3)]$ | 6. $16 \div (8 - 4) - 2$ |
| 7. $3(8 - 2) + 12$ | 8. $2[4(7 - 2) + 3]$ | 9. $6 - \frac{4 - 10}{2}$ |
| 10. $25 \div (15 \div 3) \cdot 2$ | 11. $\frac{17 - 12}{5} + (4 - 2)$ | 12. $7(12 \div 3 - 3)$ |

1-3


Writing and Evaluating Expressions

What You'll Learn

- To evaluate variable expressions
- To solve problems by evaluating expressions

... And Why

To solve real-world problems involving packaging and shopping

 **Check Skills You'll Need**
Simplify each expression.

1. $6(9 + 1)$
2. $17 - 2 + 3$
3. $9 + 8 \cdot 2 + 4$
4. $[3(5) + 1] \cdot 2$

 **for Help**
Lesson 1-2

 **New Vocabulary**
• evaluate

1 Evaluating Variable Expressions

To **evaluate** a variable expression, you first replace each variable with a number. Then, you use the order of operations to simplify.

1 EXAMPLE

Evaluate $4y - 15$ for $y = 9$.

$$\begin{aligned} 4y - 15 &= 4(9) - 15 && \text{Replace } y \text{ with } 9. \\ &= 36 - 15 && \text{Multiply.} \\ &= 21 && \text{Subtract.} \end{aligned}$$

Quick Check

1. Evaluate each expression.
 - a. $63 - 5x$, for $x = 7$
 - b. $4(t + 3) + 1$, for $t = 8$

Sometimes expressions have more than one variable.

2 EXAMPLE Replacing More Than One Variable

Evaluate $3ab + \frac{c}{2}$ for $a = 2$, $b = 5$, and $c = 10$.

$$\begin{aligned} 3ab + \frac{c}{2} &= 3 \cdot 2 \cdot 5 + \frac{10}{2} && \text{Replace the variables.} \\ &= 3 \cdot 2 \cdot 5 + 5 && \text{Work within grouping symbols.} \\ &= 6 \cdot 5 + 5 && \text{Multiply from left to right.} \\ &= 30 + 5 && \text{Multiply.} \\ &= 35 && \text{Add.} \end{aligned}$$

Quick Check

2. Evaluate each expression.
 - a. $6(g + h)$, for $g = 8$ and $h = 7$
 - b. $2xy - z$, for $x = 4$, $y = 3$, and $z = 1$
 - c. $\frac{r+s}{2}$, for $r = 13$ and $s = 11$

2 Solving Problems by Evaluating Expressions

You can write and evaluate variable expressions to solve problems.

3 EXAMPLE Real-World Problem Solving

Purchasing Energy drinks come in cases of 24 bottles.

- Write a variable expression for the number of cases a store should order to get b bottles of energy drinks.
- Evaluate the expression for 120 bottles.

a. b bottles

$$\frac{b}{24} \begin{array}{l} \leftarrow \text{bottles wanted} \\ \leftarrow \text{number in case} \end{array}$$

b. 120 bottles

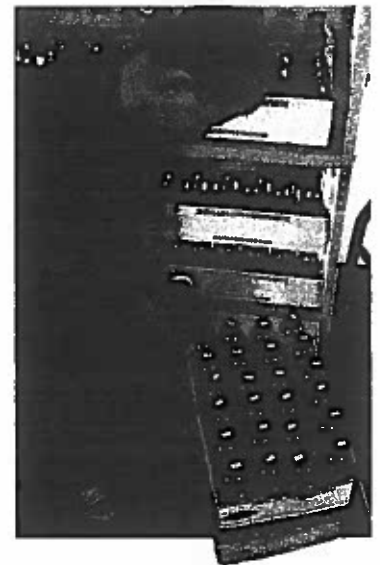
$$\frac{b}{24} = \frac{120}{24} \quad \text{Evaluate for } b = 120.$$

$$= 5 \quad \text{Divide.}$$

The store should order five cases to get 120 bottles.

Quick Check

- The store pays \$29 for each case of drinks. Write a variable expression for the cost of c cases. Find the cost of five cases.



4 EXAMPLE Real-World Problem Solving

Online Shopping An online music store charges \$14 for each CD. Shipping costs \$6 per order. Write a variable expression for the cost of ordering CDs. Find the cost of ordering eight CDs.

Table

Number of CDs	Cost of CDs	Shipping (\$)	Total Cost (\$)
1	$1 \cdot 14$	6	$1 \cdot 14 + 6 = 20$
2	$2 \cdot 14$	6	$2 \cdot 14 + 6 = 34$
4	$4 \cdot 14$	6	$4 \cdot 14 + 6 = 62$

Let n = number of CDs.

Expression $14 \cdot n + 6$

Evaluate the expression for $n = 8$.

$$14 \cdot n + 6 = 14 \cdot 8 + 6 \quad \text{Replace } n \text{ with } 8.$$

$$= 112 + 6 = 118 \quad \text{Multiply. Then add.}$$

It costs \$118 to order eight CDs.

Real-World Connection

In a case, bottles are often arranged in 4 rows of 6 (or 6 rows of 4).

Problem Solving Hint

In Example 4, the phrase *for each* implies multiplication. So \$14 for each CD means "\$14 times the number of CDs."

Quick Check

- Find the cost of ordering ten CDs.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 14)



Example 2
(page 14)

Evaluate each expression.

1. $7b$, for $b = 5$ 2. $5 - c$, for $c = 3$ 3. $x \div 8$, for $x = 40$
 4. $3n + 2$, for $n = 7$ 5. $41 - 4h$, for $h = 10$ 6. $5a + 7$, for $a = 20$

Evaluate each expression for $x = 2$, $y = 3$, and $z = 10$.

7. xyz 8. $8y \div x$ 9. $\frac{z}{5} + 2$ 10. $4y - x$
 11. $2z + xy$ 12. $\frac{9 + y}{x}$ 13. $4xy - z$ 14. $5(y + z)$

Examples 3 and 4
(page 15)

15. **Word Processing** An office assistant types 55 words per minute.
 a. Write a variable expression for the number of words the office assistant types in m minutes.
 b. Evaluate the expression for 20 minutes.
 16. **Online Purchasing** An online video store charges \$24 for each DVD. Shipping costs \$4 per order.
 a. Write a variable expression for the cost of ordering DVDs.
 b. Find the cost of ordering 3 DVDs.

B Apply Your Skills

Evaluate each expression.

17. $2a + 5$, for $a = 5$ 18. $105z$, for $z = 7$
 19. $6 \div a + 8$, for $a = 2$ 20. $19 - (a - 4)$, for $a = 8$
 21. $13ab$, for $a = 1$ and $b = 7$ 22. $16 - 4mn$, for $m = 0$ and $n = 3$
 23. $j(5 + k)$, for $j = 11$ and $k = 4$ 24. rst , for $r = 5$, $s = 5$, and $t = 5$
 25. $\frac{150}{z + y}$, for $y = 25$ and $z = 50$ 26. $\frac{x - y}{4}$, for $x = 52$ and $y = 12$



Real-World Connection

By porpoising (jumping clear of the water), dolphins can travel as fast as 26 km/h.

27. **Marine Biology** Write an expression for the number of kilometers a dolphin travels in d hours swimming at 8 km/h. Then find the number of kilometers the dolphin travels in 3 hours.
 28. **Data Analysis** Use the chart to find how many calories a 100-lb person uses in an hour of moderate walking.
 a. Write an expression for the number of calories a 100-lb person uses in moderate walking for w hours.
 b. Evaluate the expression to find the number of calories a 100-lb person uses in moderate walking for 2 hours.

Calories per Hour Used by a 100-lb Walker

Type of Walking	Calories
Slow	110
Moderate	153
Brisk	175
Racing	295

SOURCE: www.nutristrategy.com

29. **Error Analysis** Your friend evaluates $(10 - k) \div 5$ for $k = 5$ and gets 9 for an answer. Explain your friend's error.

1-4

Integers and Absolute Value

What You'll Learn

- To represent, graph, and order integers
- To find opposites and absolute values

... And Why

To represent real-world quantities that are less than zero, such as cold temperatures

Check Skills You'll Need

Write an integer for each situation.

1. lose \$7
2. find \$9
3. 8 steps forward
4. 3 yards gained
5. 5 floors down

GO for Help

Skills Handbook, p. 797

New Vocabulary

- opposites
- integers
- absolute value

1 Comparing Integers

Antifreeze is mixed with the water in a car's radiator to prevent the water from freezing. Pure water freezes at about 32 degrees Fahrenheit ($^{\circ}\text{F}$) *above* zero. A mixture of equal parts water and antifreeze freezes at about 32 degrees *below* zero.

Freezing Points

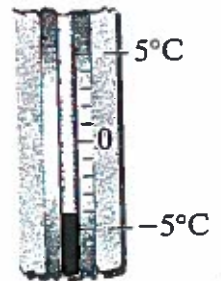
Substance	Freezing Temperature ($^{\circ}\text{F}$)
Water	32
Antifreeze and water	-32
Seawater	28
Gasoline	-36

You can write 32 degrees above zero as $+32^{\circ}\text{F}$ or 32°F . You can write 32 degrees below zero as -32°F . Read the numbers 32 and -32 as "*positive 32*" and "*negative 32*," respectively.

1 EXAMPLE Representing Negative Numbers

Temperature Write a number to represent the temperature shown by the thermometer.

The temperature of the liquid in the thermometer is 4 degrees Celsius below zero, or -4°C .

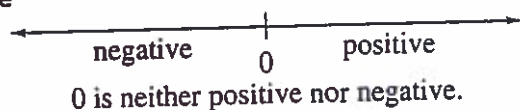


Quick Check

1. **Temperature** Seawater freezes at about 28°F , or about 2 degrees Celsius below zero. Write a number to represent the Celsius temperature.

You can graph positive and negative numbers on a number line. A number line helps you compare numbers and arrange them in order.

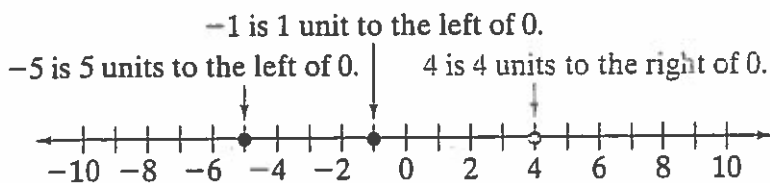
Numbers increase in value from left to right.



To compare and order numbers, you can use symbols for “is less than” ($<$), “is less than or equal to” (\leq), “is greater than” ($>$), and “is greater than or equal to” (\geq).

2 EXAMPLE Graphing on a Number Line

Graph -1 , 4 , and -5 on a number line. Compare the numbers and order them from least to greatest.



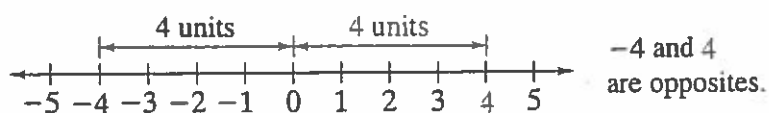
-5 is to the left of -1 , and -1 is to the left of 4 , so $-5 < -1 < 4$.
From least to greatest, the numbers are $-5, -1, 4$.

Quick Check

- Graph $0, 2$, and -6 . Compare the numbers and order the numbers from least to greatest.

2 Finding Absolute Value

Numbers that are the same distance from zero on a number line but in opposite directions are called **opposites**.



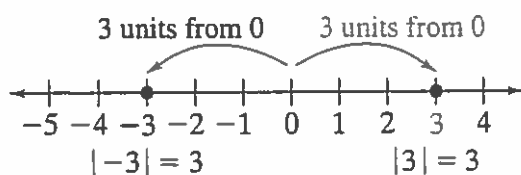
Vocabulary Tip

Recall: The **whole numbers**, $0, 1, 2, 3, 4, \dots$, are the counting numbers and zero.

Integers are the whole numbers and their opposites. A number's distance from zero on the number line is called its **absolute value**.
You write *the absolute value of 3* as $|3|$.

3 EXAMPLE Finding Absolute Value

Use a number line to find $|-3|$ and $|3|$.



Quick Check

- Write $|-10|$ in words. Then find $|-10|$.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example



Example 1
(page 18)

Write a number to represent each quantity.

- | | | |
|---------------------------|--------------------|-----------------------------|
| 1. a profit of \$250 | 2. 18°C below zero | 3. 45 s before launch |
| 4. a deposit of \$110 | 5. a debt of \$50 | 6. win by 7 points |
| 7. 300 ft below sea level | 8. a loss of 8 yd | 9. an elevation of 3,400 ft |

Example 2
(page 19)

Write the number represented by each point on the number line.



10. A 11. B 12. C

Graph each set of numbers on a number line. Then order the numbers from least to greatest.

13. -2, 8, -9 14. -3, -12, -9 15. 0, 6, -6

Example 3
(page 19)

Use a number line to find the absolute values of the integers in each pair.

16. 1, -1 17. -2, 2 18. -8, 8 19. -7, 7 20. 6, -6 21. -4, 4

Simplify each expression.

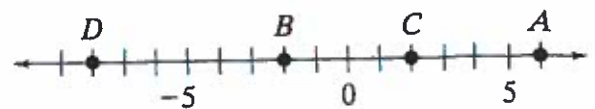
- | | |
|------------|------------------------------|
| 22. $ 18 $ | 23. the absolute value of -9 |
| 24. $ -3 $ | 25. the absolute value of 6 |
| 26. $ -7 $ | 27. the absolute value of -2 |

B Apply Your Skills

Open-Ended Describe a quantity each integer could represent.

28. -1,000 29. 28 30. -126

Write the integer represented by each point.



31. A 32. B 33. C 34. D

Simplify each expression.

- | | | |
|-------------|----------------|--------------|
| 35. $ 0 $ | 36. $ -1,000 $ | 37. $- -13 $ |
| 38. $ -56 $ | 39. $- -23 $ | 40. $- 12 $ |

Compare. Use $>$, $<$, or $=$ to complete each statement.

- | | | |
|-------------------------|--------------------------|--------------------------|
| 41. $-8 \square 0$ | 42. $4 \square -25$ | 43. $-9 \square -2$ |
| 44. $ -1 \square 50 $ | 45. $ -6 \square -12 $ | 46. $ 10 \square -10 $ |

47. **Multiple Choice** Which expression would you use to represent 10 times your height h in inches?
 (A) $10 + h$ (B) $10 - h$ (C) $10h$ (D) $10 \div h$

Write an expression to represent each quantity.

48. a loss of $\frac{1}{3}$ of an investment of d dollars
 49. n degrees Fahrenheit above $r^\circ\text{F}$ room temperature

Read the passage below before doing Exercises 50 and 51.

Finding Famous Ships

Scientist-explorer Robert D. Ballard led the expeditions that found two famous ships deep in the North Atlantic Ocean.

In 1912, the luxury passenger liner *Titanic* struck an iceberg. It came to rest 12,500 ft below sea level. *Titanic* was 882 ft long and 92 ft wide.

In 1941, the mighty warship *Bismarck* sank in battle. *Bismarck* was 823 ft long and 118 ft wide.

Star Hercules, only 269 ft long, towed the underwater camera sled that found *Bismarck* under 15,617 ft of water.

50. Write integers to represent the positions of *Titanic* and *Bismarck*.
 51. A friend says that *Bismarck*'s resting place is higher than *Titanic*'s since 15,617 is higher than 12,500. Explain your friend's error.

Complete each sentence with a word that makes it true.

52. An integer is negative, positive, or ?.
 53. All ? integers are less than zero.
 54. The opposite of a ? number is negative.
 55. The absolute value of an integer is never ?.

C Challenge

Open-Ended Name two consecutive integers between the given integers.

56. $-6, 2$ 57. $0, -4$ 58. $-8, -12$

Record Low Temperatures for Three States

State	Temperature ($^\circ\text{C}$)
California	-45
Nevada	-50
Georgia	-17

SOURCE: U.S. National Climatic Data Center

59. **a. Data Analysis** Use a number line to graph the temperatures in the chart at the left. Label each temperature with the name of the state where it was recorded.
b. Which state recorded the lowest temperature?
 60. **Writing in Math** How can you use integers to describe elevations above and below sea level?
 61. **Reasoning** Explain why $|x + y|$ and $|x| + |y|$ are not the same. Give examples to show that $|x + y| = |x| + |y|$ for some values of x and y , and $|x + y| \neq |x| + |y|$ for other values of x and y .



Test Prep

Multiple Choice

62. Which list shows the values in order from least to greatest?
A. 0, 3, -17, -25 B. -25, -17, 0, 3
C. 0, -17, -25, 3 D. -25, 0, 3, -27
63. Which expression has the value -90?
F. $|-90|$ G. 90 H. $|90|$ J. $-|90|$
64. Which list shows the values in order from least to greatest?
A. $|-6|$, 6, $|-3|$, 3 B. -6, $-|-3|$, 3, $|-6|$
C. $|-6|$, $|-3|$, $|3|$, $|6|$ D. -3, $-|-6|$, $-|3|$, 6
65. Which two integers are between -5 and 2?
F. -4, 1 G. -3, 3 H. -6, 1 J. 0, 4

Mixed Review



Lesson 1-3

Evaluate each expression.

66. $p - 5$, for $p = 19$ 67. $3d + 3$, for $d = 7$ 68. $55y$, for $y = 8$

Lesson 1-2

Compare. Use $>$, $<$, or $=$ to complete each statement.

69. $5 + 10 \div 5$ \square $(5 + 10) \div 5$
70. $(9 - 6) \div (2 + 1)$ \square $9 - 6 \div 2 + 1$

Lesson 1-1

71. Suppose you have c CDs. Your friend has 6 more CDs than you do. Write an expression for the number of CDs your friend has.

Checkpoint Quiz 1

Lessons 1-1 through 1-4

Write a variable expression for each word phrase.

1. 23 more than f 2. g divided by 34 3. product of 9 and p

Simplify each expression.

4. $17 + 16 - 13$ 5. $70 \div [5(3 + 4)]$ 6. $9 \times 6 \div 3 + 1$

Evaluate each expression for $x = 4$, $y = 6$, and $z = 12$.

7. $2x - 8$ 8. $3(z + y)$ 9. $4y - z + \frac{z}{x}$

10. **Temperature** On Monday the average temperature was -10°F . On Tuesday it was -15°F . On Wednesday it was -13°F . On Thursday it was 0°F .
a. Graph the temperatures on a number line.
b. Write the days in order from coldest to warmest.

Activity Lab

Modeling Integers

Hands-On

For Use With Lesson 1-5

You can use models, such as colored tiles, to represent integers.
 Use a yellow tile \square to represent a positive integer.
 Use a red tile \blacksquare to represent a negative integer.

1 ACTIVITY

Use models to represent the integers 3, -1, and -4.

$\square \square \square = 3$ $\blacksquare = -1$ $\blacksquare \blacksquare \blacksquare \blacksquare = -4$

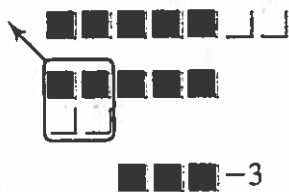
An equal number of yellow tiles and red tiles combine to make zero.

These tiles make a zero pair. $\rightarrow \square \blacksquare$ represents zero, or $\square + \blacksquare = 0$.

You can remove zero pairs in sets of mixed tiles.

2 ACTIVITY

Write the integer that is represented by $\blacksquare \blacksquare \blacksquare \blacksquare \blacksquare \square \square$.



Group the zero pairs.
Then remove them.

Write the integer that the remaining tiles represent.

EXERCISES

Use tiles to model each integer.

- | | | | |
|-------|-------|-------|-------|
| 1. -3 | 2. 5 | 3. -2 | 4. 7 |
| 5. 0 | 6. -6 | 7. 2 | 8. -8 |

Write an integer for each model.

- | | | | |
|--|---|---|---|
| 9. $\square \square$ | 10. $\blacksquare \blacksquare \blacksquare \blacksquare \blacksquare$ | 11. $\blacksquare \blacksquare$ | 12. $\square \square \square \square \square \square$ |
| 13. $\blacksquare \blacksquare \square \square$
$\square \square \blacksquare \blacksquare$ | 14. $\blacksquare \blacksquare \square \square \blacksquare \square$
\square | 15. $\square \blacksquare \square \square \square \square$
$\square \square \blacksquare \blacksquare$ | 16. $\blacksquare \blacksquare \square \square \square \square \blacksquare \blacksquare$ |

17. a. Describe how you would model the integers -15 and 25.
 b. Reasoning Suppose you combine the models from part (a). How many zero pairs could you make? How many tiles would be left after you removed the zero pairs?

1-5


Adding Integers

What You'll Learn

- To use models to add integers
- To use rules to add integers

... And Why

To use integers to solve real-world problems in sports and Earth science

 **Check Skills You'll Need**
Compare. Use $>$, $<$, or $=$ to complete each statement.

1. -6 \square -3
2. 2 \square -15
3. -5 \square $|5|$
4. $|10|$ \square $|-10|$
5. $|9|$ \square $|-2|$
6. $|-8|$ \square $|0|$

 **for Help**
Lesson 1-4

1 Using Models to Add Integers

If a car goes forward 20 ft and then backs up 20 ft, it ends where it started. Using opposite integers, you can represent this situation as $20 + (-20) = 0$.

When you add opposites, the sum is zero. So, opposites are also called *additive inverses*.

Key Concepts

Addition of Opposites

The sum of an integer and its opposite is zero.

Arithmetic

$$1 + (-1) = 0$$

$$-1 + 1 = 0$$

Algebra

$$x + (-x) = 0$$

$$-x + x = 0$$

You can use tiles to add integers. One positive tile and one negative tile combine to make a zero pair since $\square + \blacksquare = 0$.

To add integers using tiles, combine tiles and remove the zero pairs.

1 EXAMPLE Using Tiles to Add Integers

Modeling Use tiles to find $2 + (-5)$.

$$2 + (-5) \quad \square \square + \blacksquare \blacksquare \blacksquare \blacksquare \blacksquare \blacksquare \quad \text{Model the sum.}$$

$$-3 \quad \square \square \blacksquare \blacksquare \blacksquare \blacksquare \blacksquare \blacksquare \quad \text{Group and remove zero pairs. There are three negative tiles left.}$$

$$2 + (-5) = -3$$

Quick Check

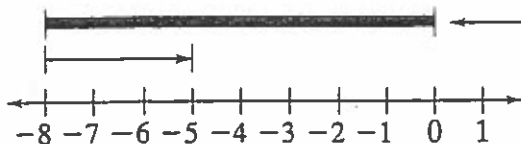
1. Use tiles to find each sum.

- a. $-1 + 4$ b. $7 + (-3)$ c. $-2 + (-2)$

A number line provides another model that you can use to add integers, as shown in Example 2.

2 EXAMPLE Using a Number Line

Football On two plays, a football team first loses 8 yd and then gains 3 yd. Find $-8 + 3$ to find the result of the two plays.



Start at 0. To represent -8 , move left 8 units. To add positive 3, move right 3 units to -5 .

$$-8 + 3 = -5$$

The result of the two plays is a loss of 5 yd.

Quick Check

2. Use a number line to find each sum.

- a. $2 + (-6)$ b. $-4 + 9$ c. $-5 + (-1)$

GO online



Video Tutor Help

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2 Using Rules to Add Integers

You can also use rules to find the sum of two integers.

Key Concepts Adding Integers

Same Sign The sum of two positive integers is positive. The sum of two negative integers is negative.

Different Signs To add two integers with different signs, find the difference of their absolute values. The sum has the sign of the integer with the greater absolute value.

3 EXAMPLE Applying Rules to Add Integers

Find each sum.

a. $-12 + (-31)$

$$-12 + (-31) = -43$$

Since both integers are negative, the sum is negative.

b. $7 + (-18)$

$$\begin{aligned} |-18| - |7| &= 18 - 7 \\ &= 11 \end{aligned}$$

Find the difference of the absolute values. Simplify.

$$7 + (-18) = -11$$

Since -18 has the greater absolute value, the sum is negative.

✓ Quick Check

3. Find each sum.

- a. $-22 + (-16)$
- b. $60 + (-13)$
- c. $-125 + 35$

4 EXAMPLE

Real-World Problem Solving

Earth Science The earthquake monitor in Hockley, Texas, is located in a salt mine at an elevation of -416 m. The elevation of the monitor in Albuquerque, New Mexico, is $2,156$ m higher than the one in Hockley. Find the elevation of the monitor in Albuquerque.

$$-416 + 2,156$$

$$|2,156| - |-416| = 2,156 - 416$$
$$= 1,740$$

$$-416 + 2,156 = 1,740$$

Write an expression.

Find the difference of the absolute values.

Simplify.

Since $2,156$ has the greater absolute value, the sum is positive.

The elevation of the monitor in Albuquerque is $1,740$ m.

✓ Quick Check

4. The elevation of a monitor in Piñon Flat, California, is $1,696$ m higher than the monitor in Hockley, Texas. Find the elevation of the monitor in Piñon Flat.

To add several integers, use the order of operations.

5 EXAMPLE

Using the Order of Operations

Find $-12 + (-6) + 15 + (-2)$.

$$-12 + (-6) + 15 + (-2) \quad \text{Add from left to right.}$$

$$\underbrace{-12 + (-6)}_{-18} + 15 + (-2)$$

The sum of two negative integers is negative.

$$\underbrace{-18 + 15}_{-3} + (-2)$$

$|-18| - |15| = 3$. Since -18 has the greater absolute value, the sum is negative.

$$\underbrace{-3 + (-2)}_{-5}$$

The sum of two negative integers is negative.

$$-12 + (-6) + 15 + (-2) = -5$$

✓ Quick Check

5. Find each sum.

- a. $1 + (-3) + 2 + (-10)$
- b. $-250 + 200 + (-100) + 220$



Real-World Connection

A worldwide network of monitors keeps track of earthquake activity. Here technicians check the monitor in Albuquerque.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

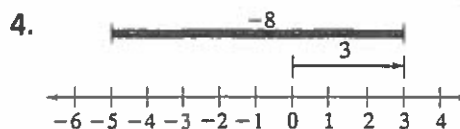
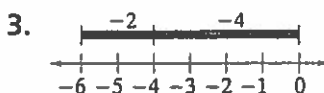
Examples 1 and 2
(pages 24 and 25)



Modeling Write an expression for each model. Find the sum.

1. + +

2. +



Draw a model and find each sum.

5. $2 + (-5)$ 6. $-5 + 2$ 7. $5 + (-2)$ 8. $-5 + (-2)$

9. $-6 + 1$ 10. $-3 + (-6)$ 11. $-3 + 2$ 12. $-3 + 4$

Example 3
(page 25)

Find each sum.

13. $14 + (-11)$ 14. $0 + (-9)$ 15. $-6 + (-7)$
 16. $-18 + 4$ 17. $-40 + 93$ 18. $-26 + (-39)$
 19. $450 + (-350)$ 20. $100 + (-100)$ 21. $235 + (-420)$

Example 4
(page 26)

22. **Geography** The highest peak at Mt. Ellsworth in Montana is 3,275 m lower than the highest peak of Mt. Kilimanjaro in Kenya, at 5,895 m. Find the elevation of the highest peak at Mt. Ellsworth.

Example 5
(page 26)

Find each sum.

23. $19 + (-9) + 45 + (-32)$ 24. $-3 + 2 + (-7) + 7 + 13$
 25. $-94 + 68 + (-22) + (-13)$ 26. $-20 + (-89) + 112 + 9$

B Apply Your Skills

Reasoning Without adding, tell whether each sum is positive, negative, or zero. Explain how you found your answer.

27. $-4 + (-10)$ 28. $11 + (-3)$ 29. $6 + (-6)$ 30. $-4 + (-2)$

Mental Math Find each sum.

31. $-5 + 20$ 32. $9 + (-9)$ 33. $10 + (-3)$ 34. $-5 + 5 + 16$

35. **Multiple Choice** One winter day the temperature starts at -12°F , warms 10 degrees, and then drops 2 degrees. What is the temperature at the end of the day?

- (A) -32°F (B) -4°F (C) -2°F (D) 0°F

Compare. Use $>$, $<$, or $=$ to complete each statement.

36. $-6 + 1$ \square $5 + 1$ 37. $0 + 3$ \square $-2 + 0$
 38. $10 + (-2)$ \square $-4 + 12$ 39. $-1 + 1$ \square $-2 + 0$
 40. $49 + (-21)$ \square $|-18|$ 41. $|-20| + (-7)$ \square $-11 + (-11)$

Evaluate each expression for $n = -15$.

42. $n + (-7) - n$ 43. $15 + n + (-8)$ 44. $n + (-15) + n$

Write a numerical expression for each of the following.
 Then find the sum.

45. You borrow \$20, and then pay back \$18.
 46. You save \$200, and then spend \$75.
 47. A man deposits \$120, and then writes a check for \$25.
 48. A submarine at 35 ft below sea level moves up 10 ft.

Use the order of operations to find each sum.

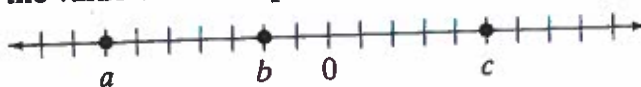
49. $4 + (-6) + 3$ 50. $-1 + 1 + (-3)$

51. $-72 + 36 + (-6) + (-18)$

52. **Football** A football team gained 4 yd, lost 2 yd, gained 11 yd, lost 8 yd, and then lost 9 yd. Find the net gain or loss.
 53. **Finance** Maria had \$123. She spent \$35, loaned \$20 to a friend, and received her \$90 paycheck. How much does she have now?
 54. **Error Analysis** A friend says that the value of $-17 + 5$ is -22 . Explain how your friend may have made this error.
 55. **Writing in Math** A friend is having trouble finding the sum of -84 and 28. What explanation would you give to help your friend?

Challenge

Reasoning For Exercises 56–59, use the number line and tell whether the value of each expression is positive or negative.



56. $a + b$ 57. $b + c$ 58. $a + a$ 59. $|a + b + c|$

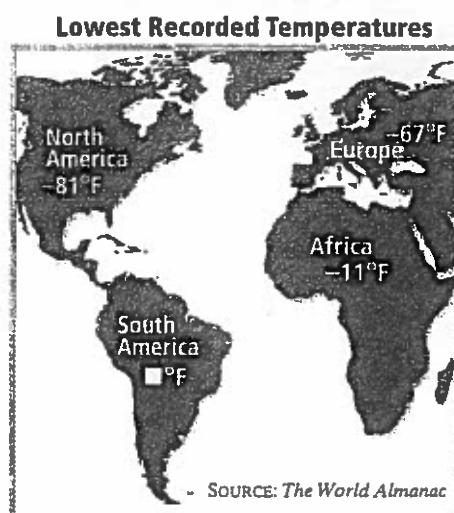


Test Prep

Multiple Choice

60. The temperature starts at -10°F , drops 2° , drops 5° , and rises 1° . Which expression gives the current temperature?
 A. $(-10) + 2 + 5 + 1$ B. $-10 + (-2) + (-5) + 1$
 C. $10 + 2 + 5 + (-1)$ D. $10 + 2 + 5 + 1$
61. A stock price starts at \$6, rises \$3, falls \$1, and falls \$1 again. What is the current price of the stock?
 F. \$1 G. \$5 H. \$7 J. \$11
62. Which statement shows an example of additive inverses?
 A. $xy = yx$ B. $x[(y + (-y))] = x(0)$
 C. $x + y = y + x$ D. $x(\frac{y}{x}) = x(1)$

63. Refer to the map at the right. The lowest temperature recorded in South America is 54 degrees higher than the lowest temperature recorded in North America. What is the lowest temperature recorded in South America?
- F. -135°F G. -27°F
 H. -17°F J. -138°F



Mixed Review



Lesson 1-4

Compare. Use $>$, $<$, or $=$ to complete each statement.

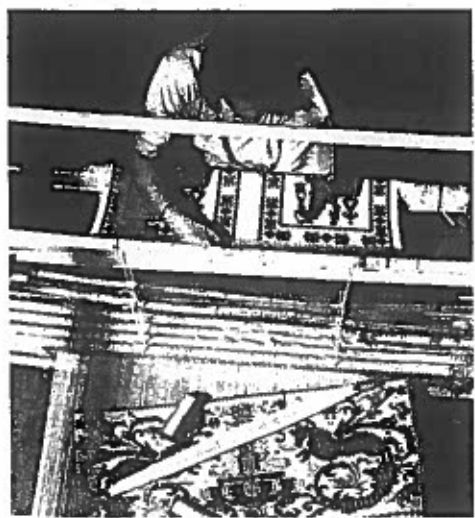
64. $-90 \square -6$ 65. $-2 \square -7$ 66. $|-15| \square -15$
 67. $0 \square -8$ 68. $-45 \square -44$ 69. $100 \square |-101|$

70. Write a numerical expression for the phrase *one hundred thirty added to the difference of one hundred sixteen and eight*. Then simplify the expression.

- Lesson 1-3 71. **Repairs** A repair center charges a \$25 flat fee plus \$10 per hour for labor. Write an expression for the cost of a repair that takes n hours. Then evaluate the expression to find the cost of an oven repair that takes 3 hours.

Math at Work

Weaver



Weaver A sturdy four-shaft floor loom, a 10-dent reed, a ski shuttle, two boat shuttles—these are some of the tools and terms of the ancient craft of weaving. Weavers use yarn, ribbon, and thread. They design and make colorful, unique items such as rugs, tapestries, and handbags. Like a pattern in algebra, each design has rules that must be followed for the desired result.

Go online

For: Information about weavers

PRSchool.com

Web Code: adb-2031

1-6

Subtracting Integers

What You'll Learn

- To use models to subtract integers
- To use a rule to subtract integers

... And Why

To use integers to solve real-world problems involving weather

Check Skills You'll Need

Find each sum.

- $8 + (-9)$
- $-11 + (-18)$
- $-4 + (-6)$
- $14 + (-3)$
- $6 + (-6)$
- $-13 + (-10)$

GO for Help
Lesson 1-5


1 Using Models to Subtract Integers

You can use tiles to help you understand subtraction of integers.

1 EXAMPLE Using Tiles to Subtract Integers

Find $-6 - (-2)$.

 Start with 6 negative tiles.

 Take away 2 negative tiles.
There are 4 negative tiles left.

$$-6 - (-2) = -4$$

Quick Check

1. Use tiles to find each difference.


- $-7 - (-2)$
- $-4 - (-3)$
- $-8 - (-5)$

You can use zero pairs to subtract an integer from a smaller integer.

2 EXAMPLE Using Zero Pairs to Subtract Integers

Find $3 - 5$.

 Start with 3 positive tiles.

 There are not enough positive tiles to take away 5. Add 2 zero pairs.

 Take away 5 positive tiles.
There are 2 negative tiles left.

$$3 - 5 = -2$$

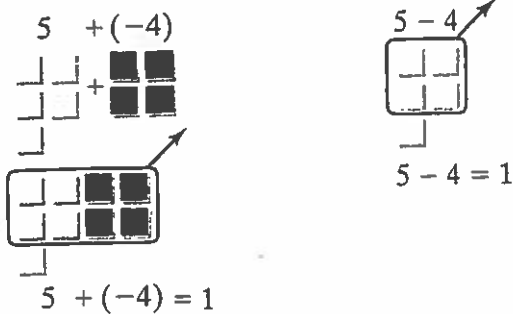
Quick Check

2. Use tiles to find each difference.

- $4 - 8$
- $-1 - 5$
- $-2 - (-7)$

2 Using a Rule to Subtract Integers

You can use models to show the relationship between adding and subtracting integers.



Both $5 + (-4)$ and $5 - 4$ equal 1. So, $5 + (-4) = 5 - 4$.

The models suggest the following rule for subtracting integers.

Key Concepts Subtracting Integers

To subtract an integer, add its opposite.

Arithmetic

$$2 - 5 = 2 + (-5) = -3$$

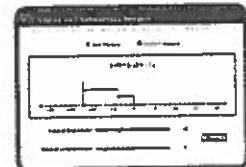
$$2 - (-5) = 2 + 5 = 7$$

Algebra

$$a - b = a + (-b)$$

$$a - (-b) = a + b$$

Online
active math



For: Integers Activity
Use: Interactive Textbook, 1-6

3 EXAMPLE Real-World Problem Solving

Weather In January, 1916, the temperature in Browning, Montana, dropped 100 degrees overnight. The initial temperature was 44°F . What was the final temperature?

$$44 - 100 \quad \text{Write an expression.}$$

$$44 - 100 = 44 + (-100) \quad \text{To subtract 100, add its opposite.}$$

$$= -56 \quad \text{Simplify.}$$

The final temperature was -56°F .

Quick Check

3. Find each difference.
- a. $32 - (-3)$ b. $-40 - 66$ c. $2 - 48$
- d. The lowest temperature ever recorded on the moon was about -170°C . The lowest temperature ever recorded in Antarctica was -89°C . Find the difference in the temperatures.



Real-World Connection

The lowest temperature ever recorded on Earth was -129°F (-89°C) in Vostok, Antarctica. Scientists there are taking ice-core samples to depths of $-3,600$ m.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Examples 1 and 2
(page 30)



Write a number sentence for each model.



Modeling Use tiles to help you find each difference.

3. $-7 - (-3)$

4. $-15 - (-7)$

5. $-5 - (-6)$

6. $-9 - (-7)$

7. $-7 - (-9)$

8. $-16 - (-9)$

9. $-8 - (-3)$

10. $-1 - (-3)$

11. $10 - (-5)$

12. $9 - (-8)$

13. $-14 - (-8)$

14. $-7 - (-7)$

15. $2 - 3$

16. $-2 - 3$

17. $-10 - 2$

Example 3
(page 31)

Write each difference as a sum. Then simplify.

18. $6 - 2$

19. $6 - (-2)$

20. $-6 - 2$

21. $2 - 6$

22. $2 - (-6)$

23. $-2 - 6$

24. $5 - 11$

25. $75 - (-25)$

26. $22 - (-7)$

27. $87 - (-9)$

28. $35 - 15$

29. $100 - (-91)$

30. **Account Balances** Terry has \$43 in a checking account. If Terry writes a check for \$62, what is the new account balance?

31. **Scores** Suppose you have a score of 35 in a game. You get a 50-point penalty. What is your new score?

B Apply Your Skills

Find each difference.

32. $-49 - 75$

33. $-65 - 15$

34. $16 - (-3)$

35. $120 - (-50)$

36. $989 - 76$

37. $-35 - 25$

38. $-92 - (-9)$

39. $-81 - (-13)$

40. $36 - 88$

Simplify.

41. $-90 - (-80) - 20$

42. $810 - 30 - (-70)$

43. $23 - (-15) - 28$

44. $-17 + 25 - (-58)$

Open-Ended Use positive and negative integers to write two different subtraction number sentences for each difference.

SAMPLE $\blacksquare - \blacksquare = -5$ $17 - 22 = -5$

$-20 - (-15) = -5$

45. $\blacksquare - \blacksquare = 0$

46. $\blacksquare - \blacksquare = 10$

47. $\blacksquare - \blacksquare = -6$

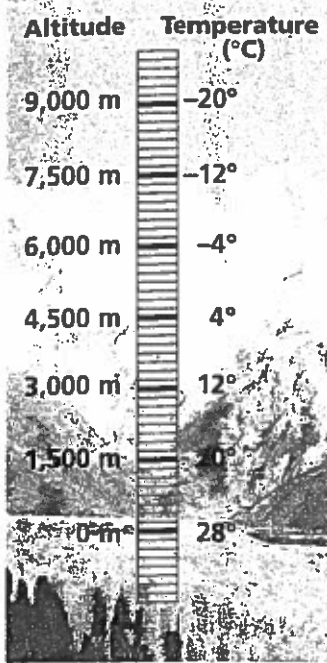
48. $\blacksquare - \blacksquare = -15$

49. $\blacksquare - \blacksquare = |-3|$

50. $\blacksquare - \blacksquare = |11|$

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Meteorology The graph at the left shows how temperature changes with altitude. Use this graph for Exercises 51–53.

51. As the altitude increases, what happens to the temperature?
 52. What is the change in temperature from 1,500 m to 6,000 m?
 53. **Multiple Choice** Find the change in temperature for every 1,500-meter increase in altitude.
 (A) 12°C (B) 8°C (C) -8°C (D) -12°C

Mental Math Simplify each expression.

54. $-6 - (-8)$ 55. $-45 - 15$ 56. $-7 - (-7) + (-7)$
 57. $100 - (-50)$ 58. $20 - (-10) - 20$ 59. $-11 + 22 - (-55)$
 60. $3 - (-3) + 6$ 61. $-32 + 2 + (-10)$ 62. $-87 + (-3) + 90$
 63. $6 - (-6) + 6$ 64. $0 + (-15) - 15$ 65. $-13 - 17 + 10$

Write a numerical expression for each phrase. Then simplify and answer the question.

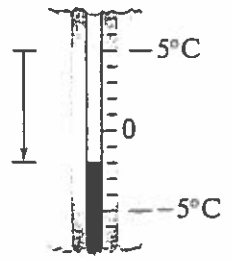
66. You are \$2 in debt. You borrow \$4 more. What is the total amount of your debt?
 67. An airplane takes off, climbs 3,000 ft, and then descends 600 ft. What is the airplane's current height?
 68. From 0°F , the temperature increases 15 degrees and then drops 25 degrees. What is the current temperature?

Challenge

Estimation Round each number. Then estimate the sum or difference.

- SAMPLE $-2,216 - 488 \approx -2,200 - 500 = -2,700$
 69. $-41 - (-86)$ 70. $-227 - 49$ 71. $-398 - 67$
 72. $-86 - 22$ 73. $288 - 59$ 74. $63 - (-21)$

75. a. **Writing in Math** A thermometer is like a vertical number line. Use the one at the right to write a subtraction problem.
 b. Write and simplify a numerical expression for your problem.



76. a. **Patterns** Copy and complete. The first one is done for you.
 $8 - (-4) = 12$
 $12 - (-4) = 16$
 $16 - (-4) = 20$
 $20 - (-4) = 24$
 $24 - (-4) = 28$
 b. If you begin at 8 and subtract -4 five times, the result is 28.
 c. Begin at 0 and subtract -4 six times. What is the result?
 77. **Reasoning** For what values of a is each statement true? Give an example, if possible.
 a. $|a - 5| = |a| - 5$ b. $|a - 5| > |a| - 5$ c. $|a - 5| < |a| - 5$

In each number square, the rows, columns, and diagonals have the same sum. Copy and complete each number square.

78.

5	-9	■
■	-1	■
-3	■	-7

sum = ■

79.

-2	■	■
-9	-5	■
-4	■	■

sum = ■

80.

■	-5	■	6
■	4	3	■
2	0	■	5
-3	■	■	-6

sum = ■

81.

-6	4	5	-9
2	■	-1	-3
-5	1	■	0
■	-7	-8	■

sum = ■



Test Prep

Multiple Choice

82. Three of the four expressions have the same value. Which one has a different value?
 A. $6 + (-4)$ B. $6 - 4$ C. $|4 - 6|$ D. $-6 - 4$
83. Suppose you have a score of 25 in a game. You get a penalty that lowers your score by 60 points. What is your new score?
 F. -85 G. -40 H. -35 J. 15
84. How many degrees warmer is a temperature of 20°C than a temperature of -7°C ?
 A. -27°C B. -13°C C. 13°C D. 27°C
85. What is the value of $-23 + -(-15) + |-17| + (-35)$?
 F. -56 G. -26 H. $|-26|$ J. 56

Mixed Review



Lesson 1-5

Find each sum.

86. $-17 + 12$

87. $-8 + 15$

88. $-9 + (-4) + 7$

Lesson 1-4

Open-Ended Complete each statement with an integer.

89. $-5 > \blacksquare$

90. $\blacksquare < 6$

91. $|-1| > \blacksquare$

92. $|\blacksquare| < 8$

Lesson 1-1

93. Write an expression for the phrase *one hundred plus the product of six and nine*. Simplify the expression.

Inductive Reasoning

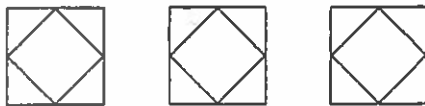
1-7

1 Writing Rules for Patterns

Inductive reasoning is making conclusions based on patterns you observe. A conclusion you reach by inductive reasoning is a **conjecture**.

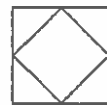
1 EXAMPLE Reasoning Inductively

Visual Patterns Use inductive reasoning. Make a conjecture about the next figure in the pattern. Then draw the figure.



Observation: The shaded triangle is rotating clockwise around the square.

Conjecture: The next figure will have a shaded triangle in the bottom-right corner.



✓ Quick Check

1. Make a conjecture about the next figure in the pattern at the right.
• Then draw the figure.



For a number pattern, a conjecture can be a rule that explains how to make the pattern. The three dots in a pattern tell you that the pattern continues.

2 EXAMPLE Writing Rules for Patterns

Number Patterns Write a rule for each number pattern.

- a. 30, 25, 20, 15, ... Start with 30 and subtract 5 repeatedly.
- b. 2, -2, 2, -2, ... Alternate 2 and its opposite.
- c. 1, 3, 4, 12, 13, ... Start with 1. Alternate multiplying by 3 and adding 1.

✓ Quick Check

2. Write a rule for each pattern.
 - a. 4, 9, 14, 19, ...
 - b. 3, 9, 27, 81, ...
 - c. 1, 1, 2, 3, 5, 8, ...

What You'll Learn

- To write rules for patterns
- To make predictions and test conjectures

... And Why

To use inductive reasoning in finding patterns and in making conjectures about economic data

✓ Check Skills You'll Need

Find each difference.

1. $-3 - 4$
2. $-7 - 4$
3. $-11 - 4$
4. $-15 - 4$

GO for Help

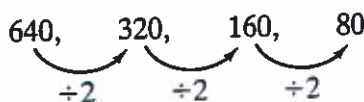
Lesson 1-6

🔊 New Vocabulary

- inductive reasoning
- conjecture
- counterexample

3 EXAMPLE Extending a Pattern

Number Patterns Write a rule for the number pattern 640, 320, 160, 80, ... Find the next two numbers in the pattern.



The first number is 640. The next numbers are found by dividing by 2.

The rule is *Start with 640 and divide by 2*. The next two numbers in the pattern are $80 \div 2 = 40$ and $40 \div 2 = 20$.

✓ Quick Check

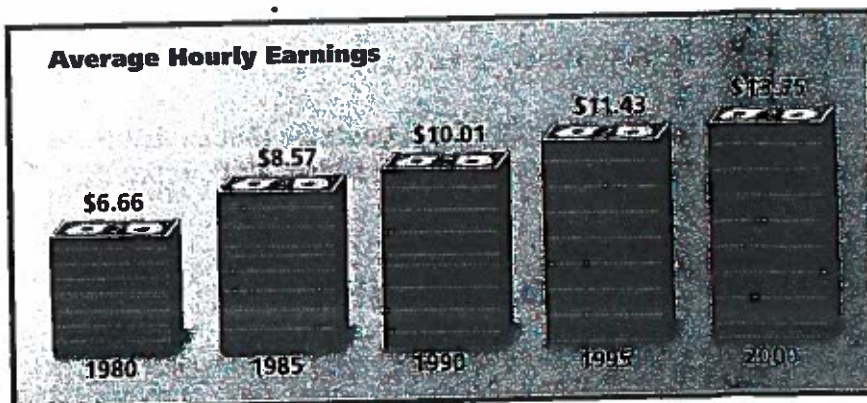
- 3. Write a rule for the pattern 1, 3, 5, 7, ... Find the next two numbers in the pattern.

2 Predictions and Counterexamples

With sufficient information, you can make predictions based on reasonable conjectures. Such predictions will probably—but not necessarily—turn out to be accurate.

4 EXAMPLE Real-World Problem Solving

Statistics See the graph below. Is a conjecture that average hourly earnings in the year 2005 will be about \$15.75 reasonable?



Average hourly earnings appear to increase by \$1.50 to \$2.50 every five years. The conjecture of \$15.75 in 2005 is reasonable, since it is about \$2.00 more than the earnings for 2000.

✓ Quick Check

- 4. You toss a coin four times, and it comes up heads each time. Is the conjecture *The coin will come up heads on every toss* reasonable? Explain.

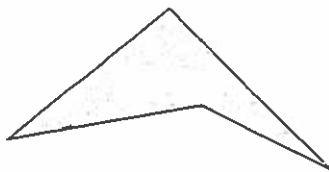
An example that proves a statement false is a **counterexample**. You need only one counterexample to prove that a conjecture is incorrect.

5 EXAMPLE Analyzing Conjectures

Inductive Reasoning Is each conjecture correct or incorrect? If it is incorrect, give a counterexample.

a. Every four-sided figure is a rectangle.

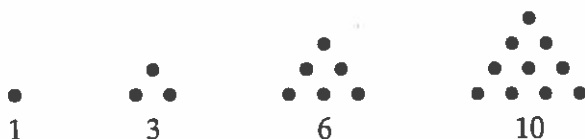
The conjecture is incorrect. The figure below has four sides, but it is not a rectangle.



b. The absolute value of any integer is positive.

The conjecture is incorrect. The absolute value of zero is zero, which is neither positive nor negative.

c. The next figure in the pattern below has 15 dots.



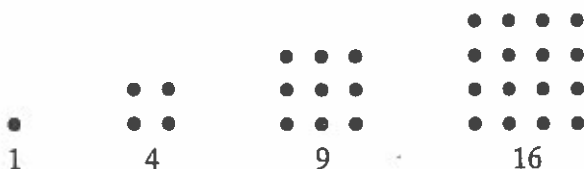
The conjecture is correct. The diagram below shows the next figure in the pattern.



✓ Quick Check

5. Is each conjecture correct or incorrect? If it is incorrect, give a counterexample.

- The last digit of the product of 5 and a whole number is either 0 or 5.
- A number and its absolute value are always opposites.
- The next figure in the pattern has 25 dots.



EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

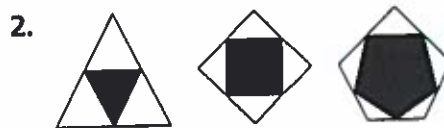
Practice and Problem Solving

A Practice by Example



Example 1
(page 35)

Visual Patterns Describe the next figure in each pattern. Then draw the figure.



Examples 2 and 3
(pages 35 and 36)

Write a rule for each pattern. Then find the next two numbers in each pattern.

3. 100, 85, 70, 55, ... 4. 5, 20, 80, 320, ... 5. 2, 7, 12, 17, ...

6. -10, -4, 2, 8, ... 7. 1, 4, 7, 10, ... 8. 1, 2, 5, 6, 9, ...

Example 4
(page 36)

9. Mario caught a cold on each of his last three visits with his cousin. Is it reasonable for Mario to conclude that his catching a cold is the result of visiting his cousin? Explain.

Example 5
(page 37)

Is each conjecture correct or incorrect? If it is incorrect, give a counterexample.

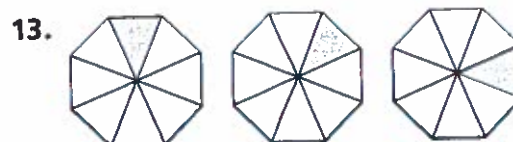
10. All birds can fly.

11. Every square is a rectangle.

12. The product of two numbers is never less than either of the numbers.

B Apply Your Skills

Visual Patterns Describe the next figure in each pattern. Then draw the figure.



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Write a rule for each pattern. Then find the next three numbers in each pattern.

15. 1, 1.5, 2, 2.5, 3, ... 16. -1, 1, -2, 2, -3, 3, ... 17. 6, 4, 2, 0, ...

Reasoning Is each conjecture correct? If incorrect, give a counterexample.

18. Every clover has three leaves.
19. The sum of two numbers is always greater than either of the two numbers.
20. A whole number is divisible by 3 if the sum of its digits is divisible by 3.

C Challenge Write a rule for each pattern. Then find the next three numbers.

21. 1, 4, 10, 22, 46, 94, ...
22. 1, -2, 4, -5, 7, -8, ...

23. a. **Writing in Math** Use the graph at the right. Write a conjecture about the unemployment rate in 2001. Justify your reasoning.
- b. How could you test your conjecture?



Test Prep

- Multiple Choice**
24. What are the next three numbers in this pattern? 11, 22, 12, 23, ...
A. 13, 23, 14 B. 13, 24, 15 C. 13, 24, 14 D. 32, 43, 33
 25. Which best describes the rule for this pattern?
1, -12, 12, -1, ...
F. subtract 24, then add 13 G. subtract 13, then add 24
H. subtract -24, then add 13 J. subtract -13, then add 24
 26. Which letter continues this pattern of letters in the alphabet?
B, E, I, N, ...
A. R B. S C. T D. U

Mixed Review



Lesson 1-6 Find each difference.

27. $1 - 8$
28. $-4 - (-9)$
29. $86 - (-17)$

Lessons 1-3, 1-5 Evaluate each expression for $x = -1$ and $y = -3$.

30. $x + y$
31. $y + x + 2$
32. $24 + x + y$

- Lesson 1-3** 33. **Science** The water in a stream flows at the rate of 1,500 gal/h. Write a variable expression for the amount of water that flows in n hours. Evaluate your expression for $n = 24$.

1-8

Problem Solving

Look for a Pattern

What You'll Learn

- To find number patterns

... And Why

To use patterns to solve real-world problems involving communication

✓ Check Skills You'll Need

Write a rule for each pattern. Find the next three numbers.

1. 8, 11, 14, 17, ...
2. 1, 5, 4, 8, 7, ...
3. 3, 5, 10, 12, 24, ...
4. 1, 4, 7, 10, ...



1

Finding Number Patterns

Math Strategies in Action

What do songs on the radio, computer code, and your body's DNA have in common?

All are based on patterns. Radio uses patterns of electromagnetic waves. Computer code consists of patterns of numbers. Your DNA is made up of molecules that repeat in special patterns.

You can solve many types of problems by finding and using patterns. Making predictions from patterns is a form of inductive reasoning.



1 EXAMPLE

Real-World Problem Solving

Information News spreads quickly at Riverdell High. Each student who hears a story repeats it 15 minutes later to two students who have not yet heard it and then tells no one else.

Suppose one student hears some news at 8:00 A.M. How many students will know the news at 9:00 A.M.?

Understand the Problem

1. How many students does each student tell?
2. How long does the news take to reach the second and third students?

Make a Plan

Make a table to organize the numbers. Then look for a pattern.

3. How many *new* students will hear the news at 8:15 A.M.?
4. How many 15-minute periods are there between 8:00 A.M. and 9:00 A.M.?

Carry Out the Plan

The pattern is to add the number of new students to the number who already know.

$$1 + 2 = 3 \quad \text{the number who know at 8:15} \\ \text{(One student talks to 2.)}$$

$$3 + 4 = 7 \quad \text{the number who know at 8:30} \\ \text{(Two students talk to 4.)}$$

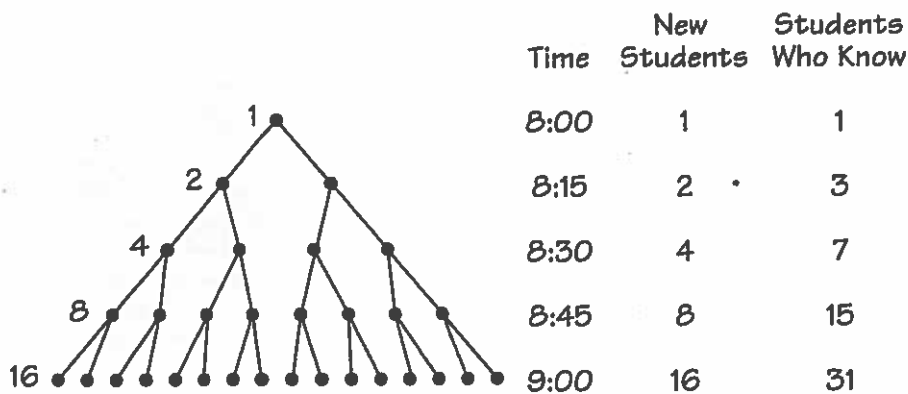
Make a table and extend the pattern to 9:00.

Time	8:00	8:15	8:30	8:45	9:00
Number of new students told	1	2	4	8	16
Number of students who know	1	$1 + 2 = 3$	$3 + 4 = 7$	$7 + 8 = 15$	$15 + 16 = 31$

By 9:00 A.M., 31 students will know the news.

Check the Answer

One way to check whether a solution is reasonable is to solve the problem by another method. You can use a *tree diagram* to show the pattern visually.



- Describe two ways to find the number of students who will know the news at 9:15 A.M.
- Suppose you want to continue the pattern beyond 9:15. Which would work better, a table or a tree diagram? Explain.
- There are 251 students at Riverdell High. By what time will every student know the news?

Quick Check

- Suppose each student who hears the story repeats it in 10 minutes. How many students will know the news at 9:00 A.M.?

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

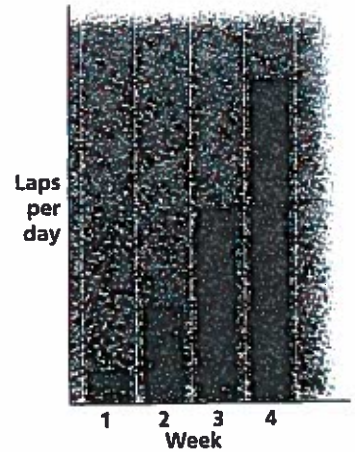
A Practice by Example

Example 1
(page 40)



Look for a Pattern to help you solve each problem.

- Data Analysis** Caroline is training for a swim meet. The graph shows the number of laps per day she swims each week. If she stays with this training pattern, how many laps per day will Caroline swim in week 8?
- Students are to march in a parade. There will be one first grader, two second graders, three third graders, and so on, through the twelfth grade. How many students will march in the parade?



- Savings** Suppose that every day you save twice as many pennies as you saved the day before. You start by saving one penny on January 1. How much money will you have in all on January 10?
- An old clock started to lose one minute each day. It was too fragile to fix, but too beloved to stop. How slow was the clock after one year of this? After two years?

B Apply Your Skills

Strategies

- Act It Out
- Draw a Diagram
- Guess, Check, Revise
- Look for a Pattern
- Make a Model
- Make a Table
- Simulate the Problem
- Solve by Graphing
- Use Multiple Strategies
- Work a Simpler Problem
- Work Backward
- Write an Equation
- Write a Proportion

Solve using any strategy.

- Geometry** You can cut a pizza into two pieces with one straight cut. With two cuts you can get four pieces. Three cuts give a maximum of seven pieces. What is the maximum number of pieces with four cuts? With five cuts?



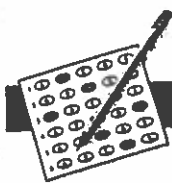
- a. Number Sense** Complete. Then look for a pattern.

$2 \cdot 2 = \square$	$3 \cdot 3 = \square$
$1 \cdot 3 = \square$	$2 \cdot 4 = \square$
Difference = \square	Difference = \square
$4 \cdot 4 = \square$	$5 \cdot 5 = \square$
$3 \cdot 5 = \square$	$4 \cdot 6 = \square$
Difference = \square	Difference = \square

- Which is greater, $10 \cdot 12$ or $11 \cdot 11$? What is the difference?
- Reasoning** Suppose you know that $47 \cdot 47 = 2,209$. Use this to find $46 \cdot 48$.
- Suppose you know that $64 \cdot 66 = 4,224$. Use this to find $65 \cdot 65$.

C Challenge

7. For a buffet dinner, a restaurant charges \$10 for one person, \$20 for two, \$29 for three, \$37 for four, \$44 for five, and so on.
- How much does a buffet dinner for 8 cost? How much does a group of 8 save by eating together rather than separately?
 - The buffet costs the restaurant \$6 per person. How large a group can the restaurant serve without losing money?
8. A woman jogging at 6 mi/h passes a man biking in the opposite direction at 12 mi/h. If they maintain their speeds, how far from each other will they be 10 minutes after passing?



Test Prep

Multiple Choice

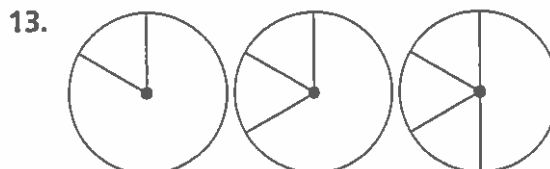
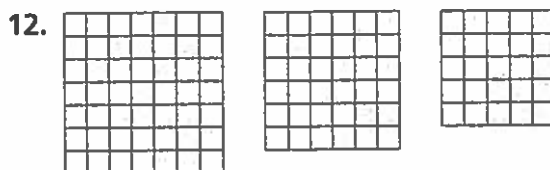
9. One edition of *Alice's Adventures in Wonderland* has 352 pages. How many 4s were used in its page numbers?
 A. 38 B. 52 C. 75 D. 88
10. Jayne has 3 quarters, 2 dimes, a nickel, and 2 pennies in her pocket. How many different amounts of money can she make using three of these coins?
 F. 24 G. 20 H. 17 J. 14
11. Assuming one yeast cell "buds" into two cells (the original cell and one new cell) at a rate of once every hour, how many yeast cells will be present from one yeast cell after 8 hours?
 A. 128 B. 256 C. 512 D. 1,024

Mixed Review



Lesson 1-7

Visual Patterns Describe the next figure in each pattern. Then draw the figure.



Lesson 1-5

Weather At midnight, the temperature was -5°F . By dawn, the temperature had risen 14° . What was the temperature at dawn?

Lesson 1-3

Evaluate each expression for $m = 1$ and $n = 4$.

15. $4m - n$ 16. $mn + 13$ 17. $4(n + 2) + m$

1-9

Multiplying and Dividing Integers

What You'll Learn

- To multiply integers using repeated addition, patterns, and rules
- To divide integers using rules

... And Why

To solve real-world problems involving deep-sea exploration and currency

Check Skills You'll Need

Simplify each expression.

1. $5 \cdot 4$ 2. $3 \cdot 8$

3. $5 \cdot 5$ 4. $14 \cdot 2$

5. $6 \cdot 5$ 6. $20 \cdot 7$



for Help

Skills Handbook, p. 781

1 Multiplying Integers

Activity

Preparing to Multiply Integers

1. Copy and complete the table. The first row is done for you.

Multiplication	Repeated Addition	Sum
$3 \cdot (-5)$	$-5 + (-5) + (-5)$	-15
$5 \cdot (-4)$	■	■
$2 \cdot (-8)$	■	■
$4 \cdot (-10)$	■	■

2. What do you notice about the signs of the sums?
3. **Inductive Reasoning** What does the pattern suggest about the product of a positive integer and a negative integer?

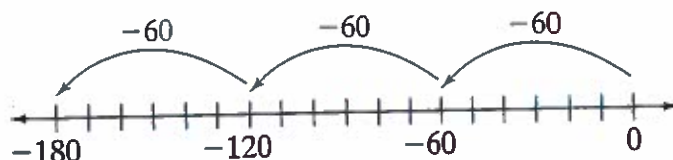
You can think of multiplication as repeated addition.

1 EXAMPLE

Real-World Problem Solving

Deep-Sea Exploration After it is launched from a boat, *Deep Rover* descends 60 ft/min. Where is it in relation to sea level 3 minutes after its launch?

Use a number line to show repeated addition.



$$3(-60) = (-60) + (-60) + (-60) = -180$$

Deep Rover is at -180 feet, or 180 feet below sea level.

Quick Check

1. Simplify each product.

• a. $2(-6)$

b. $4(-3)$

c. $7(-2)$



Real-World Connection

Scientists explore the deep waters of the Pacific Ocean in *Deep Rover*, a submersible designed for research.

You can use patterns to simplify the product of a negative number and a positive number, or the product of two negative numbers.

2 EXAMPLE Using Patterns to Multiply Integers

Patterns Use a pattern to find each product.

a. $-2(5)$

$$2(5) = 10$$

$$1(5) = 5$$

$$0(5) = 0$$

$$-1(5) = -5$$

$$-2(5) = -10$$

Start with products you know.

b. $-2(-5)$

$$2(-5) = -10$$

$$1(-5) = -5$$

$$0(-5) = 0$$

$$-1(-5) = 5$$

$$-2(-5) = 10$$

Continue the pattern.

Vocabulary Tip

Symbols for multiplication:

\times	-2×2
\cdot	$-2 \cdot 2$
$()$	$-2(3)$
$*$	$-2 * 3$

Quick Check

2. **Patterns** Use a pattern to simplify $-3(-4)$.

By inductive reasoning, the patterns from Example 2 suggest rules for multiplying integers.

Key Concepts Multiplying Integers

The product of two integers with the same sign is positive.
The product of two integers with different signs is negative.
The product of zero and any integer is zero.

Examples

$3(4) = 12$	$3(-4) = -12$
$-3(-4) = 12$	$-3(4) = -12$
$3(0) = 0$	$-4(0) = 0$

3 EXAMPLE Using Rules to Multiply Integers

Multiple Choice Which procedure is correct for multiplying $-3 \cdot 5(-4)$?

- (A) $-3 \cdot 5(-4) = 15(-4) = -60$
 (B) $-3 \cdot 5(-4) = -3(1) = -3$
 (C) $-3 \cdot 5(-4) = -3(-1) = 3$
 (D) $-3 \cdot 5(-4) = -15(-4) = 60$

Multiply from left to right. The product of a negative integer and a positive integer is negative. The product of two negative integers is positive. The product is 60. The answer is D.

Quick Check

3. Simplify each product.
- a. $-4 \cdot 8(-2)$ b. $6(-3)(5)$ c. $-7 \cdot (-14) \cdot 0$



Test-Taking Tip

Eliminate any choices that cannot be correct. Since $3 \cdot 5 \cdot 4 > 3$, you can eliminate choices B and C.

2 Dividing Integers

The rules for dividing integers are similar to those for multiplying.

Key Concepts Dividing Integers

The quotient of two integers with the same sign is positive.
The quotient of two integers with different signs is negative.
Remember that division by zero is undefined.

Examples

$$12 \div 3 = 4 \qquad 12 \div (-3) = -4$$

$$-12 \div (-3) = 4 \qquad -12 \div 3 = -4$$

4 EXAMPLE Real-World Problem Solving

Currency Find the average of the differences in the values of a Canadian dollar and a U.S. dollar for 1999–2003.

Value of Dollars (U.S. Cents)

Year	Canadian Dollar	U.S. Dollar	Difference
1999	67	100	-33
2000	67	100	-33
2001	65	100	-35
2002	64	100	-36
2003	71	100	-29

SOURCES: Bank of Canada; *The World Almanac*

$$\frac{-33 + (-33) + (-35) + (-36) + (-29)}{5} \quad \text{Write an expression for the average.}$$

$$= \frac{-166}{5} \quad \text{Use the order of operations. The fraction bar acts as a grouping symbol.}$$

$$= -33.2 \quad \text{The quotient of a negative integer and a positive integer is negative.}$$

For 1999–2003, the average difference was -33¢ . The Canadian dollar was worth an average of 33¢ less than the U.S. dollar.

Quick Check

4. Simplify each quotient.

- a. $-32 \div 8$ b. $-48 \div (-6)$ c. $-56 \div (-4)$
 d. Find the average of 4, -3, -5, 2, and -8.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

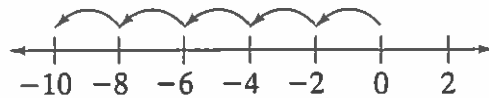
Practice and Problem Solving

A Practice by Example

Example 1
(page 44)



1. Write a number sentence for the product shown on the number line.



Write each sum as a product. Simplify the product.

2. $(-9) + (-9) + (-9) + (-9)$
3. $(-5) + (-5) + (-5) + (-5) + (-5)$

4. **Weather** The temperature dropped 5 degrees each hour for 7 h. Use an integer to represent the total change in temperature.

Simplify each product.

- | | | |
|------------------|----------------------|---------------------|
| 5. $3(-3)$ | 6. $4(-11)$ | 7. $3(-8)$ |
| 8. $5(-10)$ | 9. $6(-3)$ | 10. $2(-15)$ |
| 11. $9(-9)$ | 12. $3(-24)$ | 13. $8(-6)$ |
| 14. $-5(-3)$ | 15. $-6 \cdot 10$ | 16. $-10 \cdot 0$ |
| 17. $-9(-8)(-5)$ | 18. $0(-12) \cdot 4$ | 19. $8 \cdot 3(-4)$ |

Examples 2 and 3
(page 45)

Example 4
(page 46)

Find each quotient.

- | | | |
|---------------------|--------------------|-----------------------|
| 20. $24 \div (-24)$ | 21. $18 \div (-1)$ | 22. $-120 \div 12$ |
| 23. $56 \div (-8)$ | 24. $-72 \div 12$ | 25. $-100 \div (-10)$ |
| 26. $-38 \div (-2)$ | 27. $-72 \div 6$ | 28. $-33 \div 11$ |

For each group, find the average.

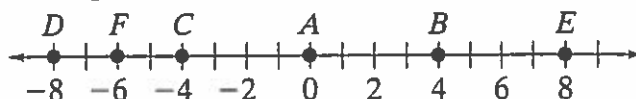
29. temperatures: -9°C , -12°C , 9°C , 4°C , -2°C
30. football yardage: 10 yd, -5 yd, 7 yd, 9 yd, -11 yd
31. golf scores: -3 , 4, 2, 1, -4 , -1 , 3, -2
32. bank balances: \$325, $-\$150$, \$130, \$200, $-\$45$

B Apply Your Skills

Mental Math Without computing, tell whether each product or quotient is *positive* or *negative*. Explain your reasoning.

33. $-6(-20)$ 34. $7(-83)$ 35. $39 \div (-3)$ 36. $-3(8)(-24)$

Name the point on the number line that is the graph of each product.



37. $-2 \cdot 0$ 38. $4(-2)$ 39. $2(-2)$ 40. $|-2| \cdot |-2|$

Use repeated addition, patterns, or rules to simplify each product or quotient.

41. $225 \div (-15)$ 42. $|-2| \cdot (-7)$ 43. $-59(-79)$
 44. $243(-88)$ 45. $-200 \div -25$ 46. $-18(-12)$
 47. $38(-2)$ 48. $1,000 \div (-50)$ 49. $24(-16)(-32)$

50. **Investing** The price of one share of a stock fell \$3 each day for 12 days.
 a. Write an integer to represent the total change in price of a share of the stock.
 b. The original stock price was \$76 per share. What was the price after the drop?

Compare. Use $>$, $<$, or $=$ to complete each statement.

51. $(-9)(-6) \blacksquare 8(-10)$ 52. $5(-2) \blacksquare (-6)(-1)$
 53. $-10 \div (-2) \blacksquare 25 \div (-5)$ 54. $-|-28| \div 7 \blacksquare -28 \div (-7)$
 55. $|-25| \div |-5| \blacksquare |-25 \div (-5)|$ 56. $-(-15 \div 5) \blacksquare -100 \div (-20)$

Number Sense Use integer rules and other math facts to answer each question.

57. What integer and -8 have the product -96 ?
 58. What integer and 9 have the product -135 ?
 59. What integer and -3 have the quotient 9 ?
 60. What two integers have a sum of negative ten and a product of negative seventy-five?

- Challenge** **Open-Ended** Simplify each pair of expressions. Then write an integer that is between the values of the expressions.

61. $-2 \cdot (-2)$ and $2 \cdot 4$ 62. $10 + (-7)$ and $10 \div (-5)$
 63. $50 + (-48)$ and $80 \div (-20)$ 64. $121 \div (-11)$ and $|-7| - |7|$
 65. a. **Inductive Reasoning** Will the sign be positive or negative for the product of three negative integers? Of four negative integers? Of five negative integers?
 b. **Writing in Math** Use inductive reasoning to write a rule for the sign of the product of more than two negative integers.
 66. **Reasoning** If a and b are positive integers, and x and y are negative integers, what is the sign of $\frac{a+b}{x+y}$? Explain.
 67. **Investing** Jerry owns 20 shares of stock valued at \$23 each. One day, the price of the stock rose \$2. It fell \$1 on each of the next three days. The stock price rose \$4 on the next day. What was the average daily gain or loss for a share of the stock over this time period? What was the total value of Jerry's stock at the end of this time period?



Test Prep

Multiple Choice

68. A scuba diver descended to a depth of 50 feet in 25 seconds. Which integer indicates the average number of feet per second the diver traveled?
 A. -50 B. -25 C. -2 D. -1
69. Which of the following is the simplest form of $\frac{-1,225}{35}$?
 F. -35 G. -25 H. -25 J. 35

In Exercises 70 and 71, what is the average for each group of data?

70. bank balances: \$200, $-\$85$, \$120, \$200, \$280
 A. \$97 B. \$119 C. \$143 D. \$177
71. feet above and below sea level:
 135 ft, -56 ft, 92 ft, -29 ft, -88 ft, -60 ft
 F. -31 ft G. -1 ft H. 19 ft J. 76 ft

Mixed Review



Lesson 1-8

72. **Reasoning** How many whole numbers from 10 to 200 have exactly two identical digits?

Lessons 1-5 and 1-6

Compare. Use $>$, $<$, or $=$ to complete each statement.

73. $-3 + (-8)$ \square $12 - (-6)$
 74. $-9 + 13$ \square $24 - 30$
 75. $|-6| - |12|$ \square $-8 + |-12|$

Lesson 1-1

Write a variable expression for each word phrase.

76. 50 decreased by a number n 77. the product of y and 60
 78. the sum of x and y 79. the quotient of d divided by 5



Checkpoint Quiz 2

Lessons 1-5 through 1-9

Simplify each expression.

1. $3 + (-11)$ 2. $12 - (-8)$ 3. $-9 \cdot 5$
 4. $-64 \div (-8)$ 5. $|3| \cdot 8 \div (-2)$ 6. $-8(-3)(3)$

Open-Ended Use integers to complete each equation.

7. $\square + \square = -7$ 8. $\square - (-20) = \square$ 9. $\square \cdot \square = -40$

Patterns Find the next three numbers in each pattern.

10. $-7, -2, 3, 8, \dots$ 11. $1, 3, 9, 27, \dots$

You can learn new vocabulary by building your own index-card word list.

- Write the term. Then write the definition.
- Include any math symbols related to the term.
- Give an example that shows how the term is used.
- Give a nonexample showing how the term might *not* apply.

EXAMPLE

Make an index card for the vocabulary term *variable expression*.

Variable Expression

Definition: A variable expression is a mathematical phrase that uses variables, numerals, and operation symbols.

Example: $h + 5$ $f - 7$ $3a$ $\frac{x}{2}$

Nonexamples: 5 $25 - 4$ 13 -21

Write the term.

Write the definition.

Give examples.

Give nonexamples.

EXERCISES

Make an index card like the one shown above for each vocabulary term. Include any helpful everyday meanings.

- | | | |
|---------------|------------------------|-------------------|
| 1. variable | 2. order of operations | 3. evaluate |
| 4. opposites | 5. integers | 6. absolute value |
| 7. conjecture | 8. inductive reasoning | 9. counterexample |

10. **Error Analysis** A student wrote the definition of *integers* at the right. Which parts are correct? Which parts are incorrect? Explain.

Integers

Definition: Integers are all the counting numbers and their opposites.

Examples: -8, 4, -1, 5, 10, -12

Nonexamples: $\frac{1}{2}$, 0, $-\frac{3}{4}$, -1.5

Activity Lab

Reading Bar and Line Graphs

Data Analysis

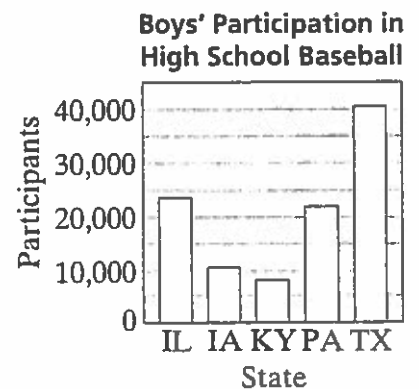
For Use With Lesson 1-10

You can analyze data presented in a graph. A bar graph usually compares quantities. The bar graph below shows boys' participation in high school baseball in several states for a recent year.

1 ACTIVITY

Use the bar graph.

1. About how many boys played high school baseball in Texas?
2. About how many boys played high school baseball in Illinois?
3. About how many more boys played high school baseball in Texas than in Illinois?
4. In which two states is the combined number of boys who played high school baseball about equal to the number who played in Pennsylvania?

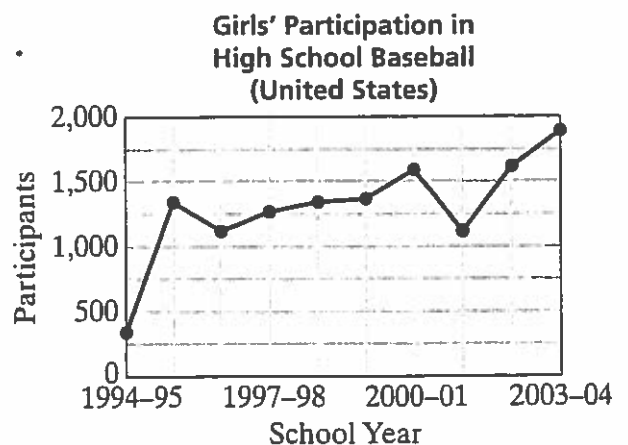


A line graph usually shows change over time. The line graph shows girls' participation in high school baseball over several years.

2 ACTIVITY

Use the line graph.

5. Between what two school years did girls' participation in high school baseball increase the most?
6. In what two school years was girls' participation in high school baseball about the same?
7. Between what two school years did girls' participation in high school baseball decrease by about 500?
8. In what school year was girls' participation in high school baseball about 1,275?
9. a. Estimate the change in participation from 1994–1995 to 2003–2004.
b. If the trend continues, how many girls will participate in high school baseball in 2014–2015?



The Coordinate Plane

1 Naming Coordinates and Quadrants

What You'll Learn

- To name coordinates and quadrants in the coordinate plane
- To graph points in the coordinate plane

... And Why

To solve real-world problems involving geography

Check Skills You'll Need

Graph the numbers on a number line.

1. $-2, 1, -5$
2. $0, 2, -4$
3. $-3, 3, -2$
4. $-1, -5, -8$

GO for Help

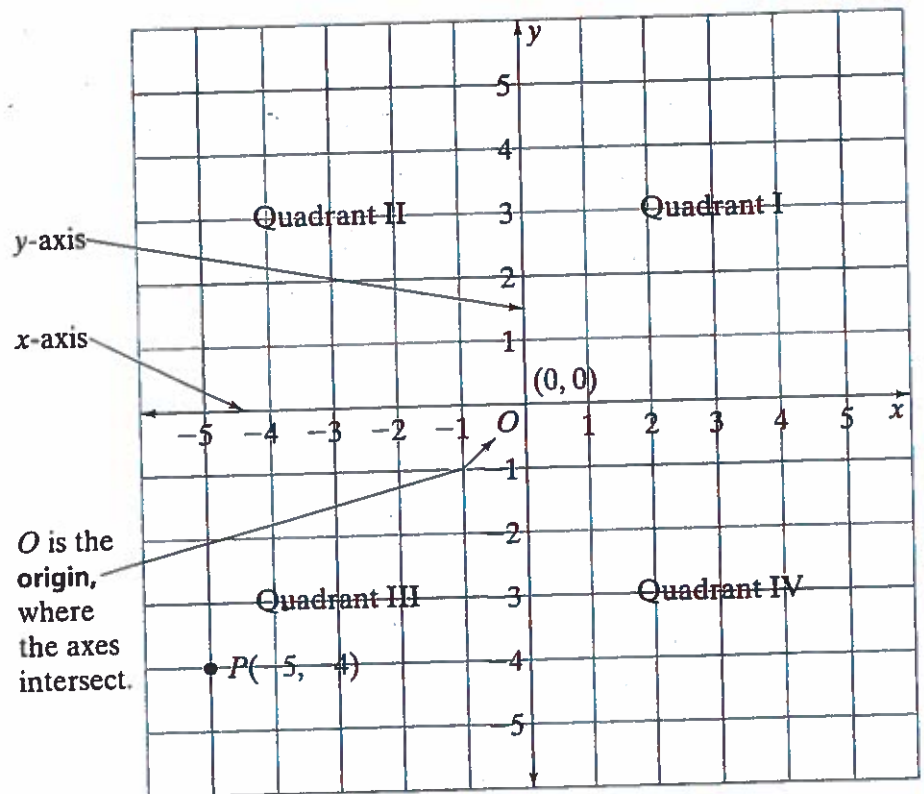
Lesson 1-4

New Vocabulary

- coordinate plane
- x-axis
- y-axis
- quadrants
- origin
- ordered pair
- x-coordinate
- y-coordinate

A **coordinate plane** is formed by the intersection of two number lines. The horizontal number line is called the **x-axis** and the vertical number line is called the **y-axis**.

The x- and y-axes divide the coordinate plane into four **quadrants**.



An **ordered pair** gives the coordinates and location of a point. The ordered pair $(-5, -4)$ identifies point P in Quadrant III above.

$(-5, -4)$

The **x-coordinate** shows the position right or left of the y-axis.

The **y-coordinate** shows the position above or below the x-axis.

Online
active math



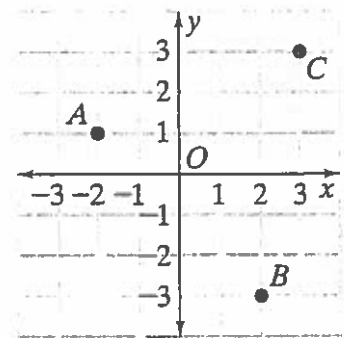
For: Coordinate Plane Activity
Use: Interactive Textbook, 1-10

1 EXAMPLE Naming Coordinates and Quadrants

Write the coordinates of point A . In which quadrant is point A located?

Point A is located 2 units to the left of the y -axis. So the x -coordinate is -2 . The point is 1 unit above the x -axis. So the y -coordinate is 1.

The coordinates of point A are $(-2, 1)$. Point A is located in Quadrant II.



Quick Check

- a. Use the graph in Example 1. Write the coordinates of B and C .
- b. Identify the quadrants in which B and C are located.

2 Graphing Points

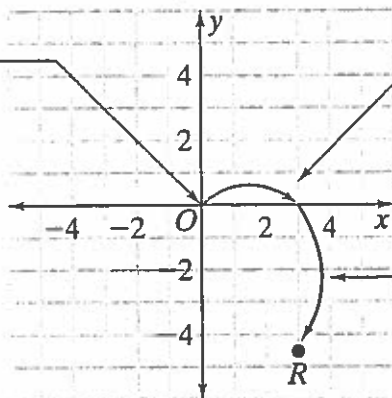
To graph a point $A(x, y)$ in a coordinate plane, you graph the ordered pair (x, y) .

2 EXAMPLE Graphing Points

Graph point $R(3, -4\frac{1}{2})$.

Step 1

Start at the origin.

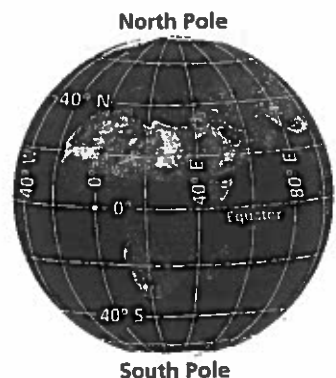


Step 2

For 3 in $R(3, -4\frac{1}{2})$, move 3 units to the right.

Step 3

For $-4\frac{1}{2}$ in $R(3, -4\frac{1}{2})$, move $4\frac{1}{2}$ units down. Draw a dot. Label it R .



Quick Check

- a. Graph these points on one coordinate plane: $K(3, 1)$, $L(-2\frac{1}{2}, 1)$, and $M(-2, -4)$.
- b. **Geometry** Draw lines to connect points K , L , and M . Describe the figure that results.

Real-World Connection

Latitude and longitude are measurements in a coordinate system that locate every point on Earth's surface.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

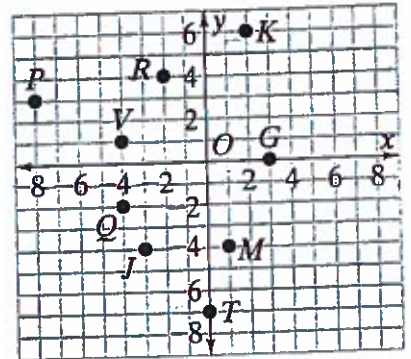
A Practice by Example

Example 1
(page 53)



In which quadrant does each point lie?

1. *J* 2. *V* 3. *M*
4. *K* 5. *P* 6. *Q*



Write the coordinates of each point.

7. *T* 8. *G* 9. *R*
10. *Q* 11. *P* 12. *M*

Example 2
(page 53)

Draw a coordinate plane. Then graph each point.

13. $A(-1, 3)$ 14. $B(-4, -1)$ 15. $C(2, 5)$ 16. $D(2, -2)$
17. $E(0, 6)$ 18. $F(-3, 2)$ 19. $G(6, 0)$ 20. $H(1, 7)$
21. $K(5, -6)$ 22. $L(0, 0)$ 23. $M(-5, -2)$ 24. $N(7, 0)$
25. $P(-1, -3)$ 26. $Q(1, 1)$ 27. $R(0, -4)$ 28. $S(-3, 4)$

B Apply Your Skills



Test-Taking Tip

You can tell in which quadrant to graph an ordered pair by looking at the signs of the coordinates.

- (+, +) goes in Q I.
- (-, +) goes in Q II.
- (-, -) goes in Q III.
- (+, -) goes in Q IV.

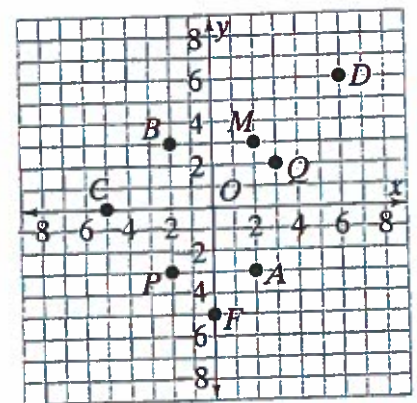
29. What ordered pair names the origin?

Name the point with the given coordinates.

30. $(3, 2)$ 31. $(0, -5)$
32. $(2, 3)$ 33. $(-2, -3)$

Write the coordinates of each point.

34. *A* 35. *B*
36. *C* 37. *D*



Mental Math Write the coordinates of each point.

38. the point 5 units to the left of the *y*-axis and 2 units below the *x*-axis
39. the point on the *y*-axis 4 units below the *x*-axis
40. **Multiple Choice** Which coordinates describe the point on the *x*-axis 3 units to the right of the origin?
(A) $(0, 3)$ (B) $(3, 0)$ (C) $(3, -3)$ (D) $(3, 3)$

Mental Math In which quadrant does $P(x, y)$ lie?

41. *x* is positive, *y* is negative. 42. *x* is positive, *y* is positive.
43. *x* is negative, *y* is positive. 44. *x* is negative, *y* is negative.

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C Challenge

Writing in Math

Describe how to graph $Q(4, 23)$ and four other points, each 1 unit from Q .

64. **Open-Ended** Draw a dot-to-dot picture on a coordinate grid. Write the coordinates of the points in order. Exchange coordinates with a classmate and draw the other's picture.
65. **Reasoning** Assume that $a \neq b$. Do (a, b) and (b, a) describe the same point? Explain.
66. Write the coordinates of four points in the coordinate plane that are 3 units from the origin. Graph the points.



Test Prep

Multiple Choice

67. $P(a, b)$ is in Quadrant III. Which word pair makes the following sentence true?
The value of a must be ? and the value of b must be ?.
- A. positive; positive B. positive; negative
C. negative; positive D. negative; negative
68. $C(x, y)$ is in Quadrant IV. Which ordered pair could be the coordinates of C ?
F. $(-3, -7)$ G. $(0, 2)$ H. $(-8, 0)$ J. $(5, -6)$
69. To graph point R , start at the origin, move 10 units to the right, 4 units down, and 6 units to the left. What are the coordinates of point R ?
A. $(4, -4)$ B. $(10, 2)$ C. $(6, 10)$ D. $(-4, -6)$
70. $P(a, b)$ is located on the x -axis. Which statement is true for all nonzero values of a ?
F. $a > b$ G. $b > a$ H. $|a| > b$ J. $|b| > a$
71. $T(a, b)$ is located in Quadrant II. Which statement is *never* true?
A. $a > b$ B. $b > a$ C. $|a| > b$ D. $|b| > a$

Mixed Review



Lesson 1-9 Find each product or quotient.

72. $-11 \cdot 11$

73. $-432 \div 48$

74. $\frac{0}{-56}$

Lesson 1-5 **75. Submarines** A submarine at sea level dives 800 ft and then another 125 ft. Find the submarine's final depth.

Lesson 1-4 Write the value of each expression.

76. $|-8|$

77. $-|-95|$

78. the opposite of 12

79. $|16| + 4$

80. $|-6| - 2$

81. the opposite of -3

Activity Lab

Graphing Ordered Pairs

Technology

For Use With Lesson 1-10

You can use a graphing calculator to display ordered pairs on a coordinate plane.

ACTIVITY

Graph these ordered pairs: $(-6, 2)$, $(-5, 6)$, $(-4, -1)$, $(-3, -5)$, $(-2, 4)$, $(0, 9)$, $(1, 5)$, $(2, -4)$, $(2, 0)$, $(3, 6)$, $(5, 2)$, $(7, -5)$, $(8, 4)$.

Step 1 Enter the ordered pairs into list L_1 for the x -coordinates and list L_2 for the y -coordinates.

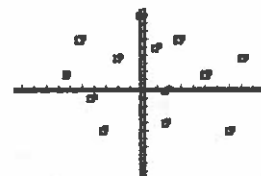
Press **LIST**. To clear old entries in L_1 , select L_1 and press **CLEAR** **ENTER**. (Clear old entries in other columns in a similar way.) Enter all the x -coordinates into list L_1 . Enter all y -coordinates into list L_2 . Check L_1 and L_2 to make sure the coordinates align as they should.

L1	L2	L3	2
1	5		
2	-4		
2	0		
3	6		
5	2		
7	-5		
8	-4		
L2(13)=4			

Step 2 In **PLOT**, enter 1 and select **On**. Select the **Type** as shown below, and check that **Xlist** and **Ylist** show L_1 and L_2 , respectively.

Plot1 On Off
 Type:
 Xlist:L1
 Ylist:L2
 Mark: + .

Step 3 Press **ZOOM** 6 to graph the data with the standard viewing window. Then press **GRAPH**.



EXERCISES

Graph each group of ordered pairs.

- $(-5, -1)$, $(-2, 4)$, $(-1, 3)$, $(0, 4)$, $(1, 6)$, $(3, 0)$, $(4, 2)$, $(5, -3)$
- $(-5, 7)$, $(4, 6)$, $(9, 2)$, $(-2, -8)$, $(0, 3)$, $(-3, -1)$, $(-6, 1)$, $(5, 5)$, $(1, -6)$, $(-1, -3)$, $(3, -2)$, $(-4, 9)$, $(-8, 4)$, $(2, -5)$, $(6, 0)$, $(7, -4)$

Graph the ordered pairs. Adjust the window settings to see all the points.

- $(-5, 2)$, $(-6\frac{1}{2}, 5)$, $(-5, -5\frac{1}{2})$, $(3, -5)$, $(-5\frac{1}{2}, -4)$, $(-4, -3\frac{1}{2})$, $(4, 5)$, $(-5\frac{1}{2}, -5)$
- $(-10, -5)$, $(-8, -9)$, $(-6, 2)$, $(-5, 8)$, $(7, -3)$, $(9, -6)$, $(10, 4)$, $(12, 9)$

Graph the ordered pairs. Describe the pattern you see.

- $(7, 3)$, $(-3, -2)$, $(1, 0)$, $(-5, -3)$, $(9, 4)$, $(3, 1)$, $(5, 2)$, $(-7, -4)$
- $(0, -5)$, $(-2, -3)$, $(4, -5)$, $(0, -1)$, $(2, -3)$, $(-4, -5)$



Some tests require you to enter a number answer on a grid. You must find the answer and also show it in a form that you can fit on the grid. For example, you can enter an improper fraction on the grid, but not a mixed number.

1 EXAMPLE

What is the sum of -0.5 and 2 ?

$$-0.5 + 2 = 1.5, \frac{3}{2}, \text{ or } 1\frac{1}{2}.$$

For the grid, you can use 1.5 or $\frac{3}{2}$, but not $1\frac{1}{2}$. You write the answer in the spaces at the top of the grid and fill in the corresponding bubbles below.

- The grids at the right are correct for 1.5 and $\frac{3}{2}$, respectively.

	1	.	5		3	/	2	
0	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Here are things to remember as you grid your responses:

- You must begin in the left column OR end in the right column.
- You cannot have blanks in the middle of a response.
- Always write a mixed number as an improper fraction or a decimal.
- You do not have to simplify fractions.

2 EXAMPLE

The surface of a lake is 29.8 ft below sea level. What is the elevation in feet of a street that is 35 ft above the surface of the lake?

$(-29.8) + 35$ Write an expression.

$|35| - |-29.8|$ Find the difference of the absolute values.

5.2 Simplify. Since 35 has the greater absolute value, the sum is positive.

	5	.	2	
0	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

The street is 5.2 feet above sea level. Enter 5.2 on the grid as shown.

- You do not enter the units.

EXERCISES

Write what you would grid for each answer.

- Simplify $12 \div [12 - (4 \cdot 2)]$.
- What is the next number in the pattern $0.4, 0.8, 1.3, 1.9, 2.6, \dots$?
- What is the next number in the pattern $0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots$?

Chapter Review

Vocabulary Review

absolute value (p. 19)
 conjecture (p. 35)
 coordinate plane (p. 52)
 counterexample (p. 37)
 evaluate (p. 14)
 inductive reasoning (p. 35)

integers (p. 19)
 opposites (p. 19)
 order of operations (p. 8)
 ordered pair (p. 52)
 origin (p. 52)
 quadrants (p. 52)

variable (p. 4)
 variable expression (p. 4)
 x-axis (p. 52)
 x-coordinate (p. 52)
 y-axis (p. 52)
 y-coordinate (p. 52)

Choose the vocabulary term that correctly completes the sentence.

- The ordered pair $(0, 0)$ represents the location of the ?.
- A letter that stands for a number in an expression is a(n) ?.
- The vertical axis in the coordinate plane is known as the ?.
- The coordinate plane is divided into four ?.
- All whole numbers and their opposites are ?.
- In the ordered pair $(-5, 2)$, the number -5 is the ?.
- The distance that a number is from zero on a number line is the ? of the number.

Go Online
 PHSchool.com

For: Vocabulary quiz
 Web Code: adj-0151

Skills and Concepts

1-1 Objectives

- ▼ To identify variables, numerical expressions, and variable expressions (p. 4)
- ▼ To write variable expressions for word phrases (p. 5)

A **variable** is a letter that stands for a number. A **variable expression** uses variables, numerals, and operation symbols.

Write a **variable expression** for each word phrase.

- | | |
|---------------------------------|-------------------------------|
| 8. twenty-five less than x | 9. the product of n and 3 |
| 10. ten decreased by t | 11. a number x divided by 4 |
| 12. a number n increased by 5 | 13. two more than y |

1-2 Objectives

- ▼ To use the order of operations (p. 8)
- ▼ To use grouping symbols (p. 9)

To simplify a numerical expression, follow the **order of operations**.

- Work inside grouping symbols.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Simplify each expression.

14. $3 \cdot 7 + 6 \div 2$ 15. $(4 + 8) \div 2 \cdot 2$ 16. $9 \cdot 5 - 4(12 \div 6)$

1-3 Objectives

- ▼ To evaluate variable expressions (p. 14)
- ▼ To solve problems by evaluating expressions (p. 15)

To **evaluate** a variable expression, substitute a number for each variable. Use the order of operations to simplify.

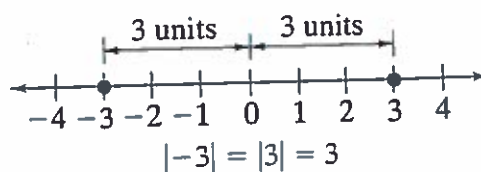
Evaluate each expression.

17. $3x + 4$, for $x = 5$ 18. $15 + 10 \div n$, for $n = 5$
19. $(y - 6)2$, for $y = 16$ 20. $4(4 + m)$, for $m = 6$
21. $15t \cdot 10$, for $t = 3$ 22. $z + [15 - (z - 1)]$, for $z = 4$

1-4 Objectives

- ▼ To represent, graph, and order integers (p. 18)
- ▼ To find opposites and absolute values (p. 19)

Integers are the set of whole numbers and their **opposites**. The **absolute value** of an integer is its distance from zero on a number line. On a number line, the integer farther to the right is the greater integer.



Simplify each expression.

23. the opposite of 17 24. $|-1,000|$
25. the absolute value of negative 9 26. the opposite of the absolute value of 12

Compare. Use $>$, $<$, or $=$ to complete each statement.

27. $-7 \blacksquare -9$ 28. $0 \blacksquare -3$ 29. $-6 \blacksquare 2$ 30. $|-5| \blacksquare |5|$
31. **Water Slides** A slide at a water park is 30 ft high. What integer represents your change in elevation when you go down the slide?

1-5 and 1-6 Objectives

- ▼ To use models to add integers (p. 24)
- ▼ To use rules to add integers (p. 25)
- ▼ To use models to subtract integers (p. 30)
- ▼ To use a rule to subtract integers (p. 31)

To add integers with the *same* sign, add their absolute values. The sum has the same sign. To add integers with *different* signs, find the difference of their absolute values. The sum has the sign of the integer with the greater absolute value. To subtract an integer, add its opposite.

Simplify each expression.

32. $8 + (-15)$ 33. $-9 + 21$ 34. $9 - (-5)$
35. $14 + (-9) + (-20)$ 36. $-62 - (-59) - 24$
37. $-7 - 4$ 38. $-4 + 12 + (-3) + (-6)$
39. **Wildlife** An eagle leaves her nest on the side of a cliff. She soars upward 60 ft and then dives 80 ft. What is her change in elevation after leaving the nest?

1-7 Objectives

- ▼ To write rules for patterns (p. 35)
- ▼ To make predictions and test conjectures (p. 36)

Inductive reasoning is making conclusions based on patterns you observe. A conclusion reached by inductive reasoning is a **conjecture**.

Write a rule for each pattern. Find the next three numbers in the pattern.

40. 0, 6, 12, 18, ... 41. -18, -9, 0, 9, ... 42. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \dots$

1-8 Objectives

- ▼ To find number patterns (p. 40)

You can use patterns to solve problems.

43. Suppose you plan to save \$12 per week. You have already saved \$7.50. In how many weeks will you have saved at least \$100?
44. A four-line classified ad costs \$28 for a week. Each additional line costs \$10.50. What is the weekly cost of a 12-line ad?

1-9 Objectives

- ▼ To multiply integers using repeated addition, patterns, and rules (p. 44)
- ▼ To divide integers using rules (p. 46)

To multiply or divide integers, multiply or divide the absolute values of the integers. If the integers have the same sign, the product or quotient is positive. If the integers have different signs, the product or quotient is negative.

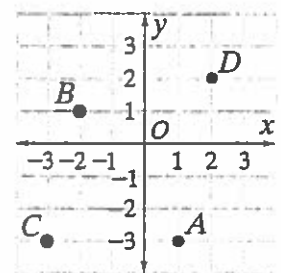
Multiply or divide.

45. $7(-6)$ 46. $250 \div (-50)$ 47. $(-9)(-8)$
48. $-56 \div (-8)$ 49. $-120 \div 40$ 50. $-15(11)$
51. $\frac{-64}{8}$ 52. $(-5)(-7)$ 53. $(-6)(-17)$

1-10 Objectives

- ▼ To name coordinates and quadrants in the coordinate plane (p. 52)
- ▼ To graph points in the coordinate plane (p. 53)

A **coordinate plane** is formed by the intersection of two number lines. The **x-axis** and the **y-axis** divide the coordinate plane into four **quadrants**. An **ordered pair** gives the coordinates of a point. The **x-coordinate** shows the position right or left of the y-axis. The **y-coordinate** shows the position above or below the x-axis.



Write the coordinates of each point.

54. A 55. B 56. C 57. D

Chapter Test

Write an expression for each phrase.

- a number n increased by nineteen
- ten less than negative three
- the product of x and negative five
- 5 more than the opposite of y

Evaluate each expression for the given values of the variables.

- $3a + 5$, for $a = -5$
- $5m + 9 + 7n$, for $m = 8$ and $n = 1$
- $3|x - y| + x$, for $x = 1$ and $y = 8$
- $20 - 2(a - b)$, for $a = 3$ and $b = 2$

Simplify each expression.

- $|-5|$
- opposite of -9
- opposite of 7
- $|15|$

Use $>$, $<$, or $=$ to complete each sentence.

- $-6 \square -5$
- $8 \square -10$
- $-3 \square 3$
- $0 \square -7$

Simplify each expression.

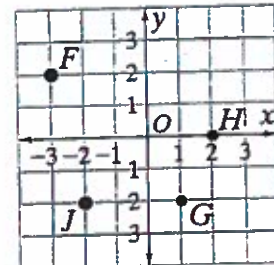
- $15 + (-7)$
- $-8 \div (+12)$
- $-9(-7)$
- $54 \div (-6)$
- $-6 \cdot 48$
- $\frac{-56}{-7}$
- $119 \div (+24)$
- $-47 + (-21)$
- $-83 + 17$
- $5(-12)(-3)(-1)$
- $2 \cdot |14 - (-9)|$
- $8 \cdot 6 \div (2 + 1)$
- $4 + 7 \cdot 2 + 8$
- $16 - 2 \cdot (5 + 3)$

In which quadrant or on which axis does each point lie?

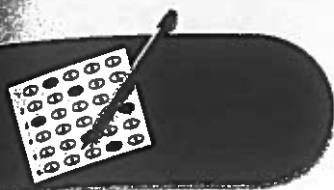
- $(-5, 7)$
- $(0, -4)$
- $(-8, -6)$

Write the coordinates of each point.

- F
- G
- H
- J



- A shirt costs \$15 and jeans cost \$25.
 - Write an expression for the cost of j jeans and s shirts.
 - Evaluate the expression to find the cost of three pairs of jeans and five shirts.
 - How many pairs of jeans can you buy for \$60?
- Which statement is *always* true?
 - The absolute value of an integer is equal to the opposite of the integer.
 - The absolute value of an integer is greater than zero.
 - An integer is greater than its opposite.
 - A positive integer is greater than a negative integer.
- A submarine was 250 m below sea level. It rose 75 m. Use an integer to describe the new depth of the submarine.
- Write a rule for the pattern below. Find the next three numbers in the pattern.
100, 90, 85, 75, 70, 60, ...
- You are in an elevator on the seventh floor. You go down 4 floors and then up 8 floors. Then you go down 3 floors and up 9 floors. The elevator goes down again 2 floors, and you get off. According to the pattern, on which floor are you now?
- Writing in Math** Describe how to order the integers 2, -6 , 9, 0, and -13 from least to greatest.



Test Prep

READING COMPREHENSION

Reading Comprehension Read each passage below. Then answer the questions on the basis of what is *stated* or *implied* in the passage.

Numbers in Nature Numbers appear everywhere in the patterns of nature. For example, the numbers of petals on flowers form patterns. Find some flowers and count their petals. You will find, with few exceptions, the number of petals to be one of 3, 5, 8, 13, 21, 34, and so forth. Also, although all snowflakes are different, each one has 6-fold symmetry. Put 20 pennies on a table and push them as close together as you can. Notice that all the pennies in the middle are surrounded by 6 others. This is an example of 6-fold symmetry.

1. What is true about the pattern for numbers of petals on flowers?
 (A) The numbers are all odd.
 (B) The increases from one to the next are always the same.
 (C) The increases from one to the next suggest a pattern you've seen before.
 (D) The number of petals on any flower has to be a number in the pattern.
2. Describe the pattern for the numbers of petals in flowers.
3. What number would follow 34 in the pattern for the numbers of petals on flowers?
 (F) 34 (G) 35 (H) 55 (J) 68
4. What do patterns in snowflakes and pennies pushed close together have in common?

Wings or Wheels? The Arctic tern, a small sea bird, is the animal that migrates the longest distance each year. It can fly from a latitude of 84°N in the Arctic to 78°S in the Antarctic, and back. For some terns, this journey may be about 25,000 mi, which is about the distance around Earth at the equator. By contrast, the average number of miles a vehicle in the United States travels each year is about 14,000. Some Arctic terns live 25 years, which means they pile up an impressive number of miles traveled in a lifetime.

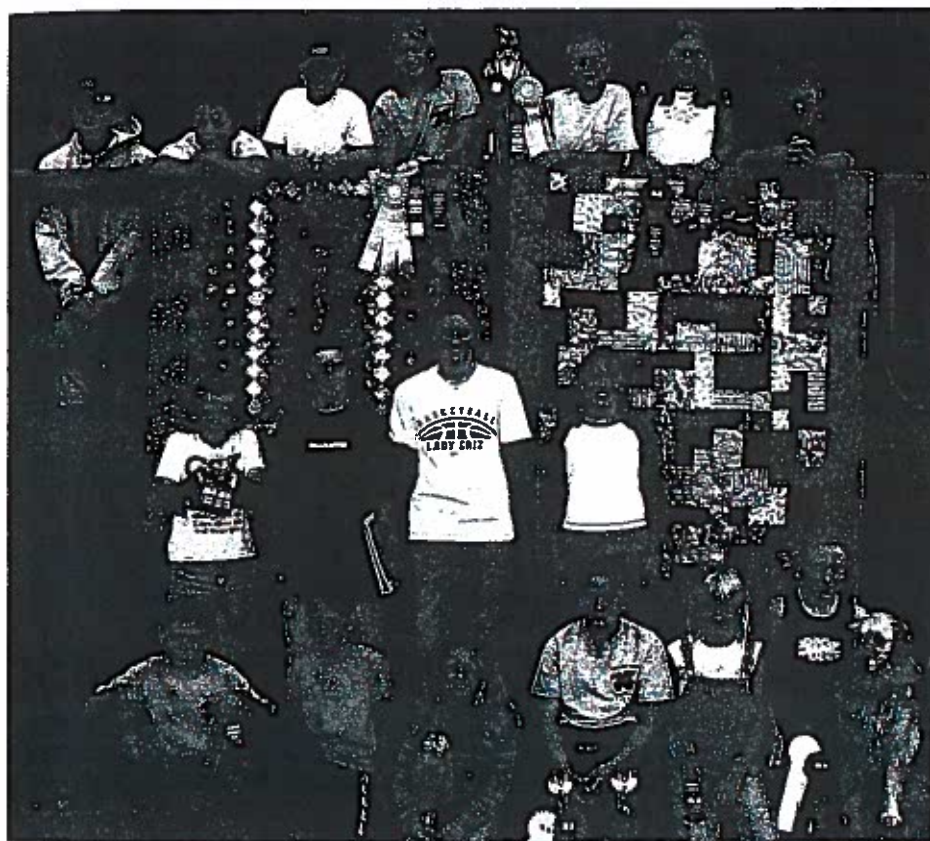
5. About how far does an Arctic tern fly in one migration south?
 (A) 162 mi (B) 14,000 mi
 (C) 12,500 mi (D) 25,000 mi
6. On average, how far does a vehicle in the United States travel each year?
 (F) 14,000 mi (G) 14,000 km
 (H) 25,000 mi (J) 25,000 km
7. Which travels farther in a year, an Arctic tern in its annual migration or a vehicle that's driven the average number of miles? About how much farther?
8. About how far could an Arctic tern fly in its migrations over 25 years? Justify your answer.

Operations With Fractions

Chapter 5

LESSONS

- 5-1 Comparing and Ordering Rational Numbers
- 5-2 Fractions and Decimals
- 5-3 Adding and Subtracting Fractions
- 5-4 Multiplying and Dividing Fractions
- 5-5 Using Customary Units of Measurement
- 5-6 Problem Solving: Work Backward
- 5-7 Solving Equations by Adding or Subtracting Fractions
- 5-8 Solving Equations by Multiplying Fractions
- 5-9 Powers of Products and Quotients



What You'll Learn Next

In this chapter, you will learn how to

- Perform operations with fractions.
- Solve equations with fractions.
- Find powers of products and quotients.
- Solve problems by working backward.



Activity Lab Applying what you learn, on pages 288–289 you will solve problems about quilting.

Key Vocabulary

- dimensional analysis (p. 258)
- least common denominator (LCD) (p. 238)
- least common multiple (LCM) (p. 236)
- multiple (p. 236)
- reciprocals (p. 254)
- repeating decimal (p. 242)
- terminating decimal (p. 241)

5-1

Comparing and Ordering Rational Numbers

What You'll Learn

- To find the least common multiple
- To compare fractions

... And Why

To solve real-world problems involving team records

Check Skills You'll Need

Write the prime factorization of each number.

1. 20 2. 125 3. 45
4. 186 5. 621 6. 1,575

GO for Help

Lesson 4-3

New Vocabulary

- multiple
- least common multiple (LCM)
- least common denominator (LCD)

1 Finding the Least Common Multiple

A **multiple** of a number is the product of that number and any nonzero whole number.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, ...

The numbers 12, 24, and 36 are *common multiples* of 4 and 6. The common multiple 12 is their **least common multiple (LCM)**.

1 EXAMPLE

Real-World Problem Solving

Sports Today, both the school baseball and school soccer teams had games. The baseball team plays every 6 days. The soccer team plays every 5 days. When will both teams have games on the same day again?

6, 12, 18, 24, 30, 36, ... List the multiples of 6.

5, 10, 15, 20, 25, 30, ... List the multiples of 5.

The LCM is 30. In 30 days both teams will have games again.

Quick Check

1. Find the LCM.

- a. 3, 4 b. 4, 5 c. 3, 4, 5

You can also use prime factorization to find the LCM.

2 EXAMPLE

Using Prime Factorization

Find the LCM of 12 and 40.

$$\left. \begin{array}{l} 12 = 2^2 \cdot \textcircled{3} \\ 40 = \textcircled{2}^3 \cdot \textcircled{5} \end{array} \right\} \text{ Write the prime factorizations.}$$

$$\begin{array}{l} \text{LCM} = 2^3 \cdot 3 \cdot 5 \\ = 120 \end{array} \quad \begin{array}{l} \text{Use the greatest power of each factor.} \\ \text{Multiply.} \end{array}$$

The LCM of 12 and 40 is 120.

Check Check

Use prime factorization to find the LCM.

- a. 16 b. 9, 15 c. 12, 15, 18

Use prime factorization to find the LCM of a variable expression.

EXAMPLE Finding the LCM of Variable Expressions

Find the LCM of $6a^2$ and $18a^3$.

$$\left. \begin{aligned} 6a^2 &= \textcircled{2} \cdot 3 \cdot a^2 \\ 18a^3 &= 2 \cdot \textcircled{3^2} \cdot \textcircled{a^3} \end{aligned} \right\} \text{Write the prime factorizations.}$$

$$\text{LCM} = 2 \cdot 3^2 \cdot a^3 \quad \text{Use the greatest power of each factor.}$$

$$= 18a^3 \quad \text{Multiply.}$$

The LCM of $6a^2$ and $18a^3$ is $18a^3$.

Check Check

Find the LCM.

- a. $12x$, $15xy$ b. $8m^2$, $14m^4$ c. $25y^2$, $15x$

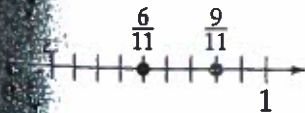
Comparing Fractions

Use a number line to compare fractions.

EXAMPLE Using a Number Line

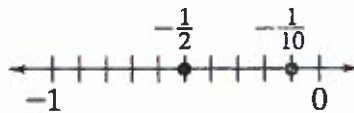
Graph and compare the fractions in each pair.

a. $\frac{6}{11}$, $\frac{9}{11}$



$\frac{9}{11}$ is on the right, so $\frac{9}{11} > \frac{6}{11}$.

b. $-\frac{1}{2}$, $-\frac{1}{10}$



$-\frac{1}{2}$ is on the left, so $-\frac{1}{2} < -\frac{1}{10}$.

online
active math



For: Fractions Activity
Use: Interactive Textbook, 5-1

Check Check

Use a number line to compare the fractions in each pair.

- a. $\frac{4}{9}$, $\frac{2}{9}$ b. $-\frac{4}{9}$, $-\frac{2}{9}$ c. $-\frac{4}{9}$, $\frac{2}{9}$

The LCD is sometimes called the **lowest common denominator**.

When fractions have different denominators, rewrite the fractions with a common denominator. Then compare the numerators. The **least common denominator (LCD)** of two or more fractions is the LCM of the denominators.

EXAMPLE Real-World Problem Solving

Academic Competitions The math team won $\frac{5}{8}$ of its competitions and the debate team won $\frac{7}{10}$ of its competitions. Which team won the greater fraction of competitions?

Step 1 Find the LCM of 8 and 10.

$$8 = 2^3 \text{ and } 10 = 2 \cdot 5$$

$$\text{LCM} = 2^3 \cdot 5 = 40$$

Step 2 Write equivalent fractions with a denominator of 40.

$$\frac{5 \cdot 5}{8 \cdot 5} = \frac{25}{40}$$

$$\frac{7 \cdot 4}{10 \cdot 4} = \frac{28}{40}$$

Step 3 Compare the fractions.

$$\frac{25}{40} < \frac{28}{40}, \text{ so } \frac{5}{8} < \frac{7}{10}.$$

The debate team won the greater fraction of competitions.



Real-World Connection

High school debate teams can have as few as 8 debates and as many as 20 debates in a school year.

Quick Check

5. Compare the fractions in each pair.

- a. $\frac{6}{7}, \frac{4}{5}$ b. $\frac{2}{3}, \frac{3}{4}$ c. $-\frac{3}{4}, -\frac{7}{10}$

EXAMPLE Ordering Fractions

Order $-\frac{1}{2}, \frac{3}{4}, -1,$ and $\frac{2}{5}$ from least to greatest.

All negative numbers are less than all positive numbers, so $-\frac{1}{2}$ and -1 are both less than $\frac{3}{4}$ and $\frac{2}{5}$. Compare each pair.

$$\left. \begin{array}{l} -1 = \frac{-1}{1} \\ \frac{-1}{1} \cdot \frac{2}{2} = \frac{-2}{2} \end{array} \right\} \begin{array}{l} \text{Change } -1 \text{ into a fraction by using } 1 \text{ as its} \\ \text{denominator. The LCM of } 1 \text{ and } 2 \text{ is } 2. \\ \text{Use } 2 \text{ as the common denominator.} \end{array}$$

$$\left. \begin{array}{l} \frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20} \\ \frac{2}{5} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20} \end{array} \right\} \begin{array}{l} \text{The LCM of } 2, 4, \text{ and } 5 \text{ is } 20. \\ \text{Use } 20 \text{ as the common denominator.} \end{array}$$

$$-\frac{2}{2} < -\frac{1}{2} \text{ and } \frac{8}{20} < \frac{15}{20}, \text{ so } -1 < -\frac{1}{2} < \frac{2}{5} < \frac{3}{4}.$$

Quick Check

6. Order from least to greatest.

- a. $\frac{2}{3}, \frac{1}{6}, 1, \frac{5}{12}$ b. $-\frac{3}{10}, \frac{1}{5}, -1, \frac{1}{2}, -\frac{7}{12}$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Find the LCM of each pair by listing multiples.

Example 1
(page 236)

- | | | | |
|-----------|----------|-----------|------------|
| 1. 10, 45 | 2. 6, 9 | 3. 12, 20 | 4. 5, 9 |
| 5. 10, 36 | 6. 7, 12 | 7. 5, 6 | 8. 5, 6, 7 |



9. **Schedules** Both the football and volleyball teams have games today. The football team plays every 7 days. The volleyball team plays every 3 days. When will both teams have games on the same day again?

Examples 2 and 3
(page 236 and 237)

Find the LCM.

- | | | | |
|----------------|---------------|------------------|----------------|
| 10. 20, 36 | 11. 15, 27 | 12. 8, 14, 20 | 13. 5, 12, 15 |
| 14. $12x, 40y$ | 15. $8x, 25y$ | 16. $2b^2, 6c^3$ | 17. $6a^3, 8a$ |

Example 4
(page 237)

Graph and compare the fractions in each pair.

- | | | | |
|--------------------------------|----------------------------------|---------------------------------|-----------------------------------|
| 18. $\frac{4}{5}, \frac{2}{5}$ | 19. $-\frac{2}{3}, -\frac{1}{3}$ | 20. $\frac{5}{8}, -\frac{5}{8}$ | 21. $\frac{11}{12}, \frac{7}{12}$ |
|--------------------------------|----------------------------------|---------------------------------|-----------------------------------|

Example 5
(page 238)

Compare the fractions in each pair.

- | | | | |
|---------------------------------------|---------------------------------------|---------------------------------------|--|
| 22. $\frac{5}{6} \square \frac{3}{4}$ | 23. $\frac{6}{8} \square \frac{7}{9}$ | 24. $\frac{1}{6} \square \frac{1}{8}$ | 25. $-\frac{5}{18} \square -\frac{1}{3}$ |
|---------------------------------------|---------------------------------------|---------------------------------------|--|

26. **Track and Field** At the track meet, Maria placed first in $\frac{4}{5}$ of her events and Carla placed first in $\frac{2}{3}$ of her events. Who placed first in the greater fraction of events?

Example 6
(page 238)

Order from least to greatest.

- | | | | |
|---|---|--|--|
| 27. $\frac{7}{9}, \frac{3}{9}, \frac{5}{9}$ | 28. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ | 29. $\frac{2}{5}, \frac{2}{3}, \frac{2}{7}, 2$ | 30. $\frac{2}{5}, -\frac{3}{8}, -\frac{1}{3}, \frac{2}{4}$ |
|---|---|--|--|

B Apply Your Skills

Mental Math Compare. Use $>$, $<$, or $=$ to complete each statement.

- | | | | |
|---|--|---|--|
| 31. $-\frac{3}{19} \square \frac{1}{200}$ | 32. $-\frac{1}{3} \square \frac{1}{3}$ | 33. $\frac{9}{11} \square \frac{7}{11}$ | 34. $-\frac{2}{7} \square \frac{4}{14}$ |
| 35. $\frac{8}{8} \square \frac{3}{3}$ | 36. $\frac{2}{10} \square \frac{2}{100}$ | 37. $\frac{2}{5} \square 3\frac{2}{5}$ | 38. $-\frac{4}{17} \square -\frac{5}{2}$ |

39. **Multiple Choice** You need $\frac{5}{8}$ yd of fabric for a craft project. You find a piece marked $\frac{2}{3}$ yd. Is the piece long enough? Explain.

- | | |
|--------------------------------------|--------------------------------------|
| (A) No; $\frac{2}{3} > \frac{5}{8}$ | (B) No; $\frac{2}{3} < \frac{5}{8}$ |
| (C) Yes; $\frac{2}{3} > \frac{5}{8}$ | (D) Yes; $\frac{2}{3} < \frac{5}{8}$ |

40. The manager of Frank's Snack Shop buys hot dogs in packages of 36. He buys hot dog buns in packages of 20. He cannot buy part of a package. What is the least number of packages of each product he can buy to have an equal number of hot dogs and buns?

Find the LCM.

- | | | | |
|------------------|--------------------|-------------------|---------------------------|
| 41. 45, 120, 150 | 42. 2, 5, 12, 15 | 43. $12x, 40$ | 44. $7ab, 8a^3b^2, 10a^4$ |
| 45. $8x, 18xy$ | 46. $9b^3, 12bc^2$ | 47. $4g^2, 10j^4$ | 48. $2x^3, 5y^2, 15xy^2$ |

Compare. Use $>$, $<$, or $=$ to complete each statement.

49. $\frac{7}{14} \square \frac{3}{6}$ 50. $-\frac{7}{9} \square -\frac{2}{3}$ 51. $\frac{8}{5} \square \frac{3}{2}$ 52. $-\frac{19}{24} \square -\frac{5}{6}$

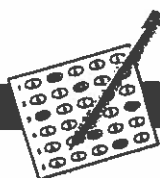
53. $-\frac{3}{8} \square -\frac{6}{16}$ 54. $\frac{10}{11} \square \frac{4}{5}$ 55. $\frac{1}{2} \square \frac{2}{4}$ 56. $-3 \square -\frac{12}{36}$

57. **Writing in Math** Jeremy and Fran want to compare $\frac{5}{8}$ to $\frac{9}{12}$. Jeremy writes equivalent fractions with a denominator of 96. Fran writes equivalent fractions with a denominator of 24. Which method would you prefer? Explain.

C Challenge

58. **Geometry** You have tiles that measure 4 in. by 5 in. What is the smallest square region you can cover without cutting or overlapping the tiles? Explain.

59. **Servings** Suppose you and your brother shared two 12-in. pizzas, a mushroom pizza cut into 8 slices and a cheese pizza cut into 6 slices. If you ate 5 slices of the mushroom pizza, and your brother ate 3 slices of the cheese pizza, who ate more pizza?



Test Prep

Multiple Choice

60. What is the LCM of 2, 3, 4, and 5?
 A. 30 B. 60 C. 90 D. 120
61. What is the GCF of $6a^3b$ and $4a^2b$?
 F. a^2b G. a^5b^2 H. $2a^2b$ J. $4a^5b^2$
62. Salt shakers come in boxes of 30 and pepper shakers come in boxes of 24. How many whole boxes of each must you buy to get an equal number of salt and pepper shakers?
 A. 4 salt, 5 pepper B. 5 salt, 4 pepper
 C. 24 salt, 30 pepper D. 30 salt, 24 pepper

Short Response

63. Are the numbers in order from least to greatest: $\frac{5}{8}, \frac{7}{16}, \frac{11}{20}$? Explain.

Mixed Review

Lesson 4-9

Write in scientific notation.

64. 5,000,000 65. 0.001394 66. 8,900,000 67. 0.000005

Lesson 4-8

Find each GCF.

68. 24, 42 69. 16, 52 70. $25c, 55c^2$ 71. $90xy, 45x^2$

Lesson 1-4

72. **History** The first modern Olympics took place in 1896 in Athens, Greece. One hundred years later, 197 nations took part in the Olympics in Atlanta, Georgia. This was 184 more nations than at the first Olympics. Solve the equation $x + 184 = 197$ to find the number of nations at the first Olympics.

Fractions and Decimals

5-2

1 Writing Fractions as Decimals

You can write a fraction as a decimal by dividing the numerator by the denominator. When the division ends with a remainder of zero, the quotient is called a **terminating decimal**.

$$\begin{array}{r} \overline{5} \text{ or } 5 \div 8 \longrightarrow 8 \overline{)5.000} \quad \leftarrow \text{quotient} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \quad \leftarrow \text{remainder} \end{array}$$

The division process for $5 \div 8$ ends with a remainder of zero. So 0.625 is a terminating decimal.

1 EXAMPLE Real-World Problem Solving

Consumer Issues A customer at a delicatessen asks for $\frac{3}{4}$ lb of potato salad. The scale reads 0.75. Is the customer getting the amount of potato salad she requested? Explain.

$$\frac{3}{4} = 3 \div 4 = 0.75$$

Since $\frac{3}{4} = 0.75$, the customer is getting the right amount of potato salad.

Quick Check

- Write each fraction or mixed number as a decimal.
 - $\frac{1}{4}$
 - $1\frac{7}{8}$
 - $3\frac{3}{10}$
 - $\frac{3}{5}$



What You'll Learn

- To write fractions as decimals
- To write terminating and repeating decimals as fractions

... And Why

To solve real-world problems involving buying food

Check Skills You'll Need

Write the decimals in order from least to greatest.

- 2.41, 0.241, 24.1, 12.4
- 1.030, 13.03, 1.300, 1.003
- 0.1, 0.01, -0.1, -0.01

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Skills Handbook, p. 784

New Vocabulary

- terminating decimal
- repeating decimal



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In a **repeating decimal**, the same block of digits repeats infinitely many times. The block of digits that repeats can be one digit or more than one digit.

2 EXAMPLE Writing a Repeating Decimal

Write each fraction as a decimal. State the block of digits that repeats.

a. $\frac{2}{3}$

$2 \div 3 = 0.66666 \dots$ ← Divide. →

$= 0.\overline{6}$

$\frac{2}{3} = 0.\overline{6}$; the digit that repeats is 6.

b. $\frac{15}{11}$

$15 \div 11 = 1.36363 \dots$

← Place a bar over the block of digits that repeats. →

$= 1.\overline{36}$

$\frac{15}{11} = 1.\overline{36}$; the block of digits that repeats is 36.

Calculator Hint
Enter $2 \div 3$ into your calculator. Then check whether the last digit in the display is 6 or 7 to see how your calculator rounds.

Quick Check

2. Write each fraction as a decimal. State whether the decimal is *terminating* or *repeating*. If the decimal repeats, state the block of digits that repeats.

a. $\frac{7}{9}$

b. $\frac{21}{22}$

c. $\frac{11}{8}$

d. $\frac{8}{11}$

When you compare and order decimals and fractions, it may be helpful to first write the fractions as decimals.

3 EXAMPLE Ordering Fractions and Decimals

Write the numbers in order, from least to greatest.

$\frac{1}{4}, -0.2, -\frac{3}{5}, 1.1$

$1 \div 4 = 0.25$
 $-3 \div 5 = -0.6$ } Change the fractions to decimals.

$-0.6 < -0.2 < 0.25 < 1.1$ Compare the decimals.

From least to greatest, the numbers are $-\frac{3}{5}, -0.2, \frac{1}{4}$, and 1.1.

Quick Check

3. Order from least to greatest.

a. $0.2, \frac{4}{5}, \frac{7}{10}, 0.5$

b. $-\frac{1}{8}, -0.75, -\frac{1}{4}, -0.375$

2 Writing Decimals as Fractions

Reading a decimal correctly provides a way to write a fraction.

Decimal	Read	Fraction
0.43	“forty-three hundredths”	$\frac{43}{100}$

If a decimal is greater than 1, you can write it as a mixed number.

4 EXAMPLE Writing a Decimal as a Fraction

Write 1.12 as a mixed number in simplest form.

$$\begin{aligned}
 1.12 &= 1\frac{12}{100} && \text{Keep the whole number 1. Write twelve hundredths as a fraction.} \\
 &= 1\frac{12 \div 4}{100 \div 4} && \text{Divide the numerator and denominator of the fraction by the GCF, 4.} \\
 1.12 &= 1\frac{3}{25} && \text{Simplify.}
 \end{aligned}$$

✓ Quick Check

4. Write as a fraction or a mixed number in simplest form.

- a. 1.75 b. 2.32 c. 0.65

You can use algebra to write a repeating decimal as a fraction.

5 EXAMPLE Writing a Repeating Decimal as a Fraction

Write the repeating decimal $0.\overline{72}$ as a fraction in simplest form.

$$\begin{aligned}
 n &= 0.\overline{72} && \text{Let the variable } n \text{ equal the decimal.} \\
 100n &= 72.\overline{72} && \text{Multiply each side by } 10^2, \text{ or } 100. \\
 100n &= 72.\overline{72} \\
 -n &= -0.\overline{72} \\
 \hline
 99n &= 72 && \text{The Subtraction Property of Equality lets you subtract the same value from each side of the equation.} \\
 \frac{99n}{99} &= \frac{72}{99} && \text{Divide each side by } 99. \\
 n &= \frac{72 \div 9}{99 \div 9} && \text{Divide the numerator and denominator by the GCF, 9.} \\
 &= \frac{8}{11} && \text{Simplify.}
 \end{aligned}$$

As a fraction in simplest form, $0.\overline{72} = \frac{8}{11}$.

Vocabulary Tip

Properties of Equality allow you to change both sides of an equation in the same way.

✓ Quick Check

5. Write each decimal as a fraction in simplest form.

- a. $0.\overline{7}$ b. $0.\overline{54}$ c. $0.\overline{213}$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example



Example 1
(page 241)

Write each fraction or mixed number as a decimal.

1. $\frac{7}{25}$ 2. $\frac{3}{5}$ 3. $1\frac{9}{20}$ 4. $6\frac{1}{4}$

5. **Remodeling** Randy and Becky measure a carpet. Becky says the carpet's length is $10\frac{5}{16}$ ft. Randy writes "10.3125 ft." Did Randy write the correct measurement? Explain.

Example 2
(page 242)

Write each fraction as a decimal. State whether the decimal is *terminating* or *repeating*. If the decimal repeats, state the block of digits that repeats.

6. $-\frac{5}{8}$ 7. $-\frac{1}{6}$ 8. $\frac{2}{9}$ 9. $\frac{9}{11}$

Example 3
(page 242)

Order from least to greatest.

10. $1.2, \frac{3}{5}, -0.5, \frac{9}{10}$ 11. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, 0.3$
 12. $-\frac{1}{4}, -\frac{1}{8}, -0.75, -0.625$ 13. $\frac{3}{2}, \frac{2}{5}, \frac{6}{5}, 0.06$
 14. $-\frac{7}{10}, -\frac{8}{10}, -0.77, -0.87$ 15. $2.1, \frac{22}{10}, 2.01, \frac{22}{11}$

Examples 4 and 5
(page 243)

Write each decimal as a fraction or a mixed number in simplest form.

16. 2.25 17. 3.4 18. 0.08 19. 7.15
 20. 2.48 21. 6.37 22. 5.36 23. 2.55
 24. $0.\bar{5}$ 25. $0.\overline{126}$ 26. $0.\overline{27}$ 27. $-0.\bar{3}$

B Apply Your Skills

Mental Math Compare. Use $>$, $<$, or $=$ to complete each statement.

28. $\frac{1}{2}$ \square 1.2 29. $\frac{7}{8}$ \square 0.875 30. $\frac{3}{5}$ \square 0.25 31. $\frac{1}{8}$ \square 0.375

32. **Number Sense** A carpenter has a bolt with diameter $\frac{5}{32}$ in. Will the bolt fit in a hole made by a drill bit with diameter 0.2 in.? Explain.

Write each fraction or mixed number as a decimal.

33. $5\frac{3}{8}$ 34. $2\frac{5}{16}$ 35. $\frac{1}{25}$ 36. $3\frac{4}{5}$ 37. $-\frac{31}{100}$ 38. $\frac{7}{11}$

Write as a fraction or a mixed number in simplest form.

39. 0.35 40. 6.8 41. -3.9 42. $10.\overline{105}$

43. **Batting averages** are usually expressed as decimals. Sarah got 32 hits in 112 times at bat. Lizzie got 26 hits in 86 times at bat.

- a. **Data Analysis** Find their batting averages to the nearest thousandth.
 b. **Probability** Based on their batting averages, who is more likely to get a hit? Explain.

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44. **Number Sense** Copy and complete this table of some commonly used fractions and decimals. Write the fractions in simplest form.

Fraction	<input type="checkbox"/>	<input type="checkbox"/>	$\frac{3}{8}$	$\frac{1}{2}$	<input type="checkbox"/>	$\frac{3}{4}$	$\frac{7}{8}$
Decimal	0.125	0.25	<input type="checkbox"/>	<input type="checkbox"/>	0.625	<input type="checkbox"/>	<input type="checkbox"/>

- C Challenge** Write as a fraction or a mixed number in simplest form.

45. $0.0\overline{6}$ 46. $0.18\overline{3}$ 47. $0.2727\overline{27}$ 48. $1.1\overline{9}$
49. **Writing in Math** Is $3.010010001\dots$ a repeating decimal? Explain.
50. **Number Sense** The number of digits that repeat in a repeating decimal is called the *period* of the decimal. The period of $0.\overline{3}$ is 1.
- Write $\frac{5}{7}$, $\frac{4}{13}$, and $\frac{7}{15}$ as decimals.
 - What is the period of each decimal you wrote in part (a)?
51. **Reasoning** Seth had just finished a division problem on his calculator when the telephone rang. He got distracted. When he looked back at the calculator, all he could see was the display 0.04040404. What might have been the division problem? Explain.



Test Prep

- Multiple Choice**
52. Which decimal is the closest approximation to $\frac{2}{3}$?
 A. 0.230 B. 0.233 C. 0.600 D. 0.667
53. A clerk puts slices of cheese on a scale until it reads 1.625 lb. What is this amount as a mixed number?
 F. $1\frac{1}{6}$ lb G. $1\frac{1}{4}$ lb H. $1\frac{5}{8}$ lb J. $1\frac{6}{25}$ lb
- Short Response**
54. Lucia's math teacher asks her to write $\frac{3}{11}$ as a decimal. She enters $3 \div 11$ on her calculator. The calculator displays 0.2727273. (a) Is this the answer Lucia should record? (b) Explain your response.

Mixed Review



- Lesson 5-1** Order the fractions in each group from least to greatest.

55. $-\frac{1}{3}, \frac{2}{3}, -\frac{5}{6}, \frac{1}{6}$ 56. $\frac{5}{8}, \frac{3}{8}, \frac{1}{5}, \frac{3}{5}, \frac{1}{8}$ 57. $-\frac{4}{7}, -\frac{1}{14}, -\frac{3}{14}, -\frac{6}{7}$

- Lesson 4-9** 58. **Geography** Lake Mead, located between Arizona and Nevada, has a capacity of $34,850,000,000 \text{ m}^3$. Write this number in scientific notation.



- Lessons 3-5 and 3-6** Solve each equation.

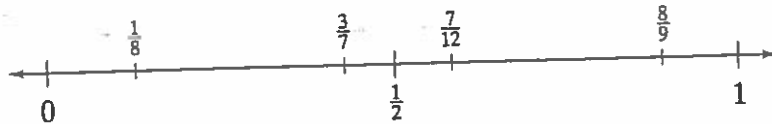
59. $12.9 + x = 27.3$ 60. $9.4m = -32.9$

Activity Lab

Estimating With Fractions and Mixed Numbers

For Use With Lesson 5-3

You can round to estimate sums and differences involving fractions and mixed numbers. In one method, you round the fraction or the fraction part of a mixed number to 0 , $\frac{1}{2}$, or 1 .



Round to 0 when the numerator is less than half of the denominator.

Round to $\frac{1}{2}$ when the numerator is about half the denominator.

Round to 1 when the numerator and denominator are almost equal.

EXAMPLE

Shopping Eva bought $\frac{2}{3}$ lb of Swiss cheese, $\frac{1}{4}$ lb of jack cheese, and $\frac{3}{8}$ lb of cheddar. About how much did she buy altogether?

$$\frac{2}{3} \approx \frac{1}{4} = 0 \quad \frac{3}{8} \approx \frac{1}{2} \quad \leftarrow \text{Round each fraction.}$$

$$0 + 0 + \frac{1}{2} = 1\frac{1}{2} \quad \leftarrow \text{Add.}$$

Eva bought about $1\frac{1}{2}$ lb of cheese.

You can get reasonable estimates when multiplying by first rounding to the nearest whole number. For division, use compatible numbers.

EXAMPLE

a. Estimate $4\frac{1}{8} \cdot 1\frac{9}{10}$.

$$\begin{array}{r} 4\frac{1}{8} \cdot 1\frac{9}{10} \\ \downarrow \quad \downarrow \\ 4 \cdot 2 = 8 \end{array} \quad \begin{array}{l} \text{If the fractional part is} \\ \text{greater than } \frac{1}{2}, \text{ round up.} \\ \\ \text{Multiply.} \end{array}$$

b. Estimate $16\frac{1}{5} \div 2\frac{3}{4}$.

$$\begin{array}{r} 16\frac{1}{5} \div 2\frac{3}{4} \\ \downarrow \quad \downarrow \\ 15 \div 3 = 5 \end{array} \quad \begin{array}{l} 2\frac{3}{4} \text{ rounds to 3. A number} \\ \text{compatible with 3 and} \\ \text{close to } 16\frac{1}{5} \text{ is 15.} \\ \\ \text{Divide.} \end{array}$$

EXERCISES

Estimate the value of each expression.

1. $\frac{2}{3} + \frac{7}{8}$

2. $5\frac{1}{12} - 2\frac{7}{9}$

3. $\frac{1}{5} + 5\frac{5}{8}$

4. $4\frac{11}{24} - \frac{7}{12}$

5. $\frac{11}{12} \cdot 4$

6. $6\frac{8}{9} \div 1\frac{1}{5}$

7. $10\frac{1}{10} \div 4\frac{7}{8}$

8. $2\frac{4}{5} \cdot 5$

8. **Shopping** Ira bought $\frac{5}{8}$ lb roast beef and $\frac{3}{4}$ lb of turkey. About how much more turkey did he buy than roast beef?

Adding and Subtracting Fractions

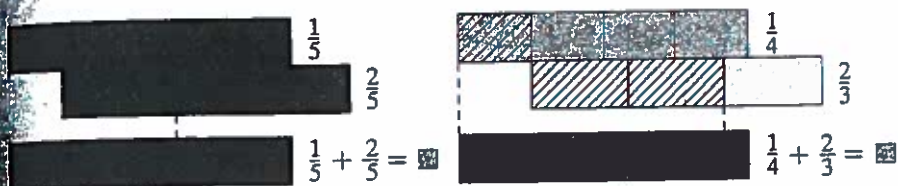
5-3

1 Adding and Subtracting Fractions

Activity

Using Models to Add Fractions

Use the models to answer each question below.



1. a. Refer to the model for $\frac{1}{5} + \frac{2}{5}$. What fraction does $\frac{1}{5} + \frac{2}{5}$ equal?
 - b. **Reasoning** The sum of any two fractions can be written as a fraction. Write a conjecture about how to find the numerator and denominator of such a sum.
2. a. Refer to the model for $\frac{1}{4} + \frac{2}{3}$. What fraction does $\frac{1}{4} + \frac{2}{3}$ equal?
 - b. Can you add the numerators to find the sum? Explain.
 - c. Can you add the denominators to find the sum? Explain.

In the model for $\frac{1}{5} + \frac{2}{5}$ above, you can see that the sum (or difference) of fractions with the same denominator is the sum (or difference) of the numerators. The denominators do not change.

1 EXAMPLE Simplifying With Like Denominators

Find each sum or difference. Simplify if possible.

a. $\frac{1}{8} + \frac{3}{8}$

b. $\frac{9}{x} - \frac{7}{x}$

$$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8}$$

Add or subtract the numerators.

$$\frac{9}{x} - \frac{7}{x} = \frac{9-7}{x}$$

$$= \frac{4}{8} = \frac{1}{2}$$

Simplify.

$$= \frac{2}{x}$$

Quick Check

1. Find each sum or difference. Simplify if possible.

a. $\frac{3}{7} + \frac{1}{7}$

b. $\frac{2}{k} + \frac{3}{k}$

c. $\frac{7}{10} - \frac{3}{10}$

d. $\frac{11}{y} + \left(-\frac{5}{y}\right)$

What You'll Learn

- To add and subtract fractions
- To add and subtract mixed numbers

... And Why

To solve real-world problems involving cooking

Check Skills You'll Need

Find the LCM of each group of numbers or expressions.

1. 4, 8
2. 9, 18
3. $2n$, 5
4. 3, 6, 9
5. 8, 5, 4
6. 10, n

GO for Help
Lesson 5-1

Before you can add or subtract fractions with unlike denominators, first write the fractions with a common denominator. The method shown here for addition works with subtraction also.

Arithmetic

$$\frac{2}{3} + \frac{1}{5}$$

$$\frac{2}{3} \cdot \frac{5}{5} + \frac{1}{5} \cdot \frac{3}{3}$$

$$\frac{10}{15} + \frac{3}{15}$$

$$\frac{13}{15}$$

Algebra

$$\frac{a}{b} + \frac{c}{d}$$

$$\frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b}$$

$$\frac{ad}{bd} + \frac{bc}{bd}$$

$$\frac{ad + bc}{bd}$$

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active math



For: Fractions Activity
Use: Interactive Textbook, S-3

2 EXAMPLE Simplifying With Unlike Denominators

Simplify each difference.

a. $\frac{1}{8} - \frac{5}{6}$

$$\frac{1}{8} - \frac{5}{6} = \frac{1 \cdot 6 - 8 \cdot 5}{8 \cdot 6}$$

$$= \frac{6 - 40}{48}$$

$$= \frac{-34}{48} = -\frac{17}{24}$$

Use a common denominator.

Use the Order of Operations.

Simplify.

b. $\frac{1}{8} - \frac{5x}{6}$

$$\frac{1}{8} - \frac{5x}{6} = \frac{1 \cdot 6 - 8 \cdot 5x}{8 \cdot 6}$$

$$= \frac{6 - 40x}{48}$$

Quick Check

2. Find each sum or difference. a. $-\frac{7}{8} + \frac{3}{4}$ b. $\frac{3}{7} - \frac{2}{m}$



Real-World Connection

Mt. Shasta, in northern California, is 14,162 ft high. You can hike the Avalanche Gulch route from Bunny Flat for 6 mi and climb 7,000 ft.

2 Adding and Subtracting Mixed Numbers

Before you add or subtract mixed numbers, write the mixed numbers as improper fractions.

3 EXAMPLE Real-World Problem Solving

Multiple Choice Ali hiked $2\frac{2}{3}$ mi near Mt. Shasta and then another $1\frac{3}{4}$ mi to his campsite. How far did he hike in all?

- (A) $\frac{11}{12}$ mi (B) $1\frac{5}{12}$ mi (C) $3\frac{11}{12}$ mi (D) $4\frac{5}{12}$ mi

$$2\frac{2}{3} + 1\frac{3}{4} = \frac{8}{3} + \frac{7}{4}$$

$$= \frac{8 \cdot 4 + 3 \cdot 7}{3 \cdot 4}$$

$$= \frac{32 + 21}{12}$$

$$= \frac{53}{12} = 4\frac{5}{12}$$

Write mixed numbers as improper fractions.

Rewrite using a common denominator.

Use the Order of Operations to simplify.

Write as a mixed number.

Ali hiked $4\frac{5}{12}$ mi in all. The answer is D.

Quick Check

3. Find each sum or difference. Simplify if possible.

a. $5\frac{3}{4} + \frac{7}{8}$ b. $5\frac{2}{3} - 3\frac{1}{6}$ c. $2\frac{3}{8} + \frac{7}{8}$

d. A recipe for punch calls for $1\frac{1}{2}$ qt of orange juice, $1\frac{1}{4}$ qt of ginger ale, and $\frac{3}{4}$ qt of cranberry juice. How many quarts of punch will the recipe make?

You can subtract mixed numbers in more than one way.

More Than One Way

You are making banana bread for a bake sale, using the recipe at the right. You have $1\frac{3}{4}$ c of sugar left in a bag of sugar. How much more sugar do you need?

Banana Bread



5 ripe bananas	$3\frac{1}{2}$ cups flour
4 eggs	2 tsp baking soda
1 cup shortening	1 tsp salt
$2\frac{1}{2}$ cups sugar	$1\frac{1}{2}$ cups chopped walnuts, optional
3 tsp vanilla	

Tina's Method

You write both mixed numbers as improper fractions.

$$\begin{aligned} 2\frac{1}{2} - 1\frac{3}{4} &= \frac{5}{2} - \frac{7}{4} \\ &= \frac{5 \cdot 4 - 2 \cdot 7}{2 \cdot 4} \\ &= \frac{20 - 14}{8} \\ &= \frac{6}{8} = \frac{3}{4} \end{aligned}$$

You need $\frac{3}{4}$ c more sugar.



Kevin's Method

You write $2\frac{1}{2}$ as $2\frac{2}{4}$, and then rewrite it as $1\frac{6}{4}$ before subtracting.

$$\begin{aligned} 2\frac{1}{2} - 1\frac{3}{4} &= 2\frac{2}{4} - 1\frac{3}{4} \\ &= 1\frac{6}{4} - 1\frac{3}{4} \\ &= \frac{3}{4} \end{aligned}$$

You need $\frac{3}{4}$ c more sugar.



Test-Taking Tip

To add or subtract fractions they must have the same denominator. Sometimes you may only have to rename one fraction, but sometimes you may need to rename both.

Choose a Method

1. For the problem above, which method do you prefer? Explain.
2. Which method would you use to find $2\frac{4}{7} - 1\frac{9}{14}$? Which method would you use to find $-1\frac{1}{2} - 1\frac{3}{4}$? Explain your choices.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 247)



Find each sum or difference. Simplify if possible.

1. $\frac{3}{16} + \frac{7}{16}$ 2. $\frac{6}{z} + \left(-\frac{2}{z}\right)$ 3. $\frac{15}{q} - \frac{8}{q}$ 4. $\frac{5}{11} + \frac{4}{11}$

5. $\frac{11}{12} - \frac{7}{12}$ 6. $\frac{7}{8} + \frac{5}{8}$ 7. $\frac{3}{10} - \frac{7}{10}$ 8. $\frac{2}{x} + \frac{3}{x}$

Example 2
(page 248)

Simplify each sum or difference.

9. $\frac{3}{4} - \frac{2}{3}$ 10. $\frac{12}{20} - \frac{1}{4}$ 11. $-\frac{3}{10} - \frac{5}{100}$ 12. $\frac{6}{x} - \frac{2}{5}$

Example 3
(page 248)

Find each sum or difference. Simplify if possible.

13. $3\frac{3}{4} + 2\frac{1}{4}$ 14. $\frac{4}{16} + 1\frac{3}{8}$ 15. $10\frac{1}{8} + 3\frac{3}{4}$ 16. $3\frac{5}{8} + 2\frac{7}{12}$

17. $1\frac{5}{9} - 1\frac{2}{9}$ 18. $5\frac{3}{4} - 2\frac{1}{8}$ 19. $1\frac{17}{18} - \frac{7}{9}$ 20. $1\frac{7}{8} - 2\frac{3}{4}$

21. **Homework** Kim works on Social Studies homework for $2\frac{2}{5}$ h. Then she works on Math homework for $1\frac{1}{4}$ h. How many hours total does Kim spend doing homework?

B Apply Your Skills

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Estimation Estimate each sum or difference.

22. $2\frac{1}{3} + 7\frac{1}{8}$ 23. $25\frac{5}{18} - 9\frac{11}{17}$ 24. $15\frac{3}{4} + 31\frac{1}{2}$ 25. $-4\frac{7}{8} + 15\frac{1}{10}$

26. **Writing in Math** Describe why estimating a sum or difference before adding or subtracting is useful.

27. **Crafts** A doll artist cuts a piece of lace $8\frac{5}{8}$ in. long from a piece $10\frac{1}{2}$ in. long. How many inches of lace are left?

Find each sum or difference.

28. $\frac{12}{15} + \frac{1}{2}$ 29. $\frac{3}{n} - \frac{3}{10}$ 30. $\frac{7}{10} + \frac{2d}{3}$ 31. $\frac{5}{6} + \frac{7}{9}$

Mental Math Find each sum.

32. $\frac{3}{4} + \frac{3}{8} + \frac{1}{4}$ 33. $2\frac{5}{7} + 1\frac{2}{5} + 3\frac{2}{7}$ 34. $\frac{2}{7} + \frac{x}{2} + \left(-\frac{2}{7}\right)$

35. **Weather** There were three snowstorms last winter. The storms dropped $3\frac{1}{2}$ in., $6\frac{1}{2}$ in., and $10\frac{3}{4}$ in. of snow. What was the combined snowfall of the three storms?

C Challenge Use prime factors to find the LCD. Then simplify each expression.

36. $\frac{7}{24} - \frac{15}{90}$ 37. $\frac{-5}{66} + \frac{-7}{99}$ 38. $\frac{2}{28} + \frac{1}{49}$

39. **Collections** Dora and Paul have a collection of x marbles. Dora has $\frac{x}{3}$ marbles. What fraction of the marbles does Paul have?



Test Prep

Multiple Choice

40. Which sum or difference is greater than 0?

- A. $-\frac{7}{8} + \frac{3}{4}$ B. $-\frac{7}{8} - \frac{3}{4}$ C. $-\frac{7}{8} + (-\frac{3}{4})$ D. $\frac{7}{8} + (-\frac{3}{4})$

41. Sue is fishing. She catches a bass weighing $5\frac{1}{4}$ lb. Then she catches three more weighing $3\frac{1}{2}$ lb, $1\frac{3}{4}$ lb, and 2 lb. She releases the smallest fish. What is the total weight of the fish Sue keeps?

- F. $8\frac{3}{4}$ lb G. $9\frac{1}{4}$ lb H. $10\frac{3}{4}$ lb J. $12\frac{1}{2}$ lb

42. Which expression is equal to $\frac{1}{3} + \frac{1}{6}$?

- A. $\frac{1}{2} + \frac{2}{4}$ B. $\frac{1}{4} + \frac{2}{8}$ C. $\frac{1}{5} + \frac{2}{10}$ D. $\frac{1}{7} + \frac{2}{14}$

Short Response

43. In 2003, first-class postage in the United States costs 37¢ for 1 oz. Your letter weighs $\frac{3}{4}$ oz. (a) Do you need extra postage to include a newspaper clipping that weighs $\frac{3}{8}$ oz? (b) Explain.

44. José and Letty plan to ride their bicycles at least eight miles. They ride for $5\frac{3}{8}$ miles and stop for a break. Then they ride for another $2\frac{5}{7}$ miles. (a) Do they meet their goal? (b) Explain your answer.

Mixed Review



Lesson 5-2

Order from least to greatest.

45. $\frac{5}{8}, \frac{4}{7}, \frac{3}{6}$

46. $\frac{2}{3}, 0.6, 0.66$

47. $\frac{10}{9}, \frac{9}{10}, -\frac{9}{10}, -\frac{10}{9}$

Lesson 4-7

Simplify each expression.

48. $x \cdot x^2$

49. $(x^3)^4$

Lesson 3-3

50. **Data Analysis** Use the data at the right. Find the mean, median, and mode of the annual salaries. Which statistic would you use to encourage someone to take a job at Company A?

10 Salaries at Company A

\$26,000	\$62,000
\$30,000	\$22,000
\$22,000	\$26,000
\$50,000	\$21,000
\$22,000	\$65,000

Checkpoint Quiz 1

Lessons 5-1 through 5-3

Compare. Use $>$, $<$, or $=$ to complete each statement.

1. $\frac{2}{3} \square \frac{2}{5}$

2. $2\frac{2}{3} \square 2\frac{4}{6}$

3. $-\frac{1}{5} \square -\frac{1}{8}$

4. $-1.65 \square -1\frac{5}{8}$

Write each fraction or mixed number as a decimal and each decimal as a fraction in simplest form.

5. $\frac{51}{100}$

6. 0.012

7. $1\frac{1}{4}$

8. $0.\bar{3}$

9. $\frac{5}{6}$

10. $0.\overline{51}$

Find each sum or difference. Simplify if possible.

11. $\frac{6}{13} + \frac{5}{13}$

12. $\frac{11}{12} - \frac{7}{9}$

13. $1\frac{3}{5} + 2\frac{7}{8}$

14. $4\frac{1}{7} - 3\frac{10}{21}$

5-4

Multiplying and Dividing Fractions

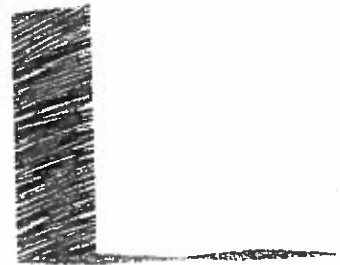
1 Multiplying Fractions

Activity

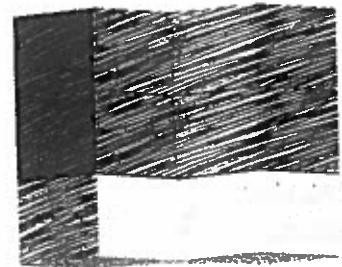
Modeling Multiplication of Fractions

Use paper folding to find $\frac{2}{3}$ of $\frac{1}{4}$, or $\frac{2}{3} \cdot \frac{1}{4}$.

1. Fold a sheet of paper into fourths as shown. Shade $\frac{1}{4}$ of it.



2. Now unfold the paper and fold it into thirds as shown in the second picture. Shade $\frac{2}{3}$ of it.



3. a. Count the small rectangles.
b. How many did you shade twice?
c. What fraction of the small rectangles is this?

4. Use your model to complete:

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{\square}{\square}$$

5. **Modeling** Use paper folding and shading to find $\frac{3}{4} \cdot \frac{1}{2}$.

To multiply fractions, first multiply their numerators and multiply their denominators. Then write the result in simplest form.

1 EXAMPLE Multiplying Fractions

Find $\frac{3}{7} \cdot \frac{4}{5}$. Simplify if possible.

$$\begin{aligned} \frac{3}{7} \cdot \frac{4}{5} &= \frac{3 \cdot 4}{7 \cdot 5} && \longleftarrow \text{Multiply the numerators.} \\ &= \frac{12}{35} && \longleftarrow \text{Multiply the denominators.} \\ &= \frac{12}{35} && \text{Simplify.} \end{aligned}$$

Quick Check

1. Find each product. Simplify if possible.

a. $\frac{2}{5} \left(\frac{1}{3} \right)$

b. $-\frac{5}{6} \cdot \frac{2}{3}$

c. $\frac{7}{8} \cdot \frac{5}{9}$

d. $-\frac{1}{4} \left(-\frac{3}{8} \right)$

What You'll Learn

- To multiply fractions
- To divide fractions

... And Why

To solve real-world problems involving area

Check Skills You'll Need

Write each mixed number as an improper fraction.

1. $2\frac{1}{3}$ 2. $3\frac{3}{10}$ 3. $1\frac{4}{9}$

4. $4\frac{4}{5}$ 5. $7\frac{7}{8}$ 6. $5\frac{1}{7}$

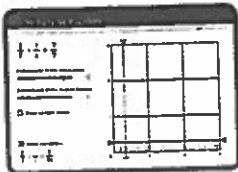
GO for Help

Skills Handbook, p. 794

New Vocabulary

- reciprocals

Online
active math



For: Fractions Activity
Use: Interactive Textbook, 5-4

When a numerator and a denominator have common factors, you can simplify before multiplying.

2 EXAMPLE Simplifying Before Multiplying

a. Find $\frac{9}{15} \cdot \frac{-5}{9}$.

$$\frac{9}{15} \cdot \frac{-5}{9} = \frac{1\cancel{9}}{3\cancel{15}} \cdot \frac{-\cancel{1}5}{\cancel{1}9} \quad \text{Divide the common factors.}$$

$$= -\frac{1}{3} \quad \text{Multiply.}$$

b. Find $\frac{y}{4} \cdot \frac{8}{11}$.

$$\frac{y}{4} \cdot \frac{8}{11} = \frac{y}{1\cancel{4}} \cdot \frac{\cancel{8}^2}{11} \quad \text{Divide the common factors.}$$

$$= \frac{2y}{11} \quad \text{Multiply.}$$

Problem Solving Hint

If either the numerator or denominator is negative, or if the negative sign is in front of the fraction, then the entire fraction is negative. $\frac{-17}{24} = -\frac{17}{24}$

Quick Check

2. Find each product. Simplify if possible.

a. $\frac{2}{3} \cdot \frac{6}{7}$ b. $-\frac{5}{15} \cdot \frac{21}{25}$ c. $\frac{2x}{9} \cdot \frac{3}{4}$

To multiply mixed numbers, first write them as improper fractions. Then simplify before multiplying, if possible.

3 EXAMPLE Real-World Problem Solving

Geometry Central Park in New York City is a rectangle. It is approximately $2\frac{1}{2}$ mi long and $\frac{1}{2}$ mi wide. What is the area of Central Park?

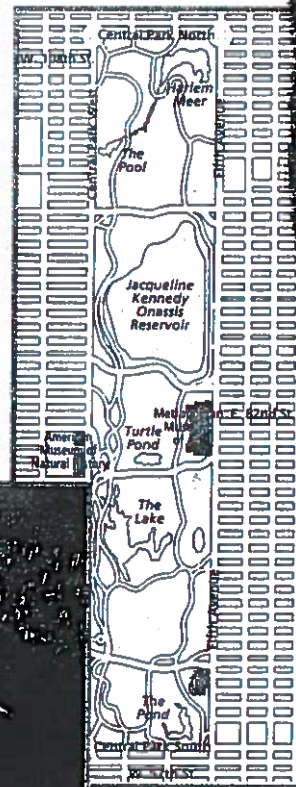
$$A = 2\frac{1}{2} \cdot \frac{1}{2} \quad \text{Area of a rectangle} = \text{length} \cdot \text{width.}$$

$$= \frac{5}{2} \cdot \frac{1}{2} \quad \text{Write } 2\frac{1}{2} \text{ as an improper fraction, } \frac{5}{2}.$$

$$= \frac{5}{4} \quad \text{Multiply.}$$

$$= 1\frac{1}{4} \quad \text{Write as a mixed number.}$$

The area of Central Park is about $1\frac{1}{4}$ mi².



Real-World Connection

Central Park is a rectangle. The angle of the photo makes two sides appear to be not parallel.

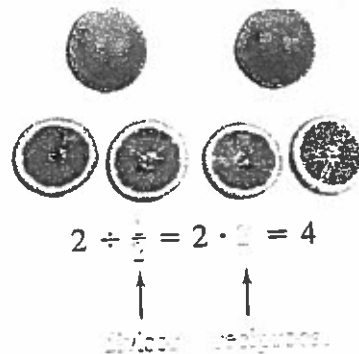
Quick Check

3. Find each product. Simplify if possible.

a. $3\frac{3}{4} \cdot \frac{2}{5}$ b. $\frac{2}{3} \cdot 1\frac{2}{7}$ c. $(-2\frac{5}{6}) \cdot 1\frac{3}{5}$

2 Dividing Fractions

Asking “What is $2 \div \frac{1}{2}$?” is the same as asking “How many halves are in two wholes?” As the oranges show, there are four halves in two wholes.



Vocabulary Tip

Reciprocals are also called **multiplicative inverses**.

Numbers like $\frac{1}{2}$ and 2 (or $\frac{2}{1}$) are **reciprocals** because their product is 1. To divide fractions, rewrite the division as a related multiplication in which you multiply by the reciprocal of the divisor.

EXAMPLE Dividing Fractions

a. Find $\frac{2}{9} \div \frac{2}{5}$.

$$\begin{aligned} \frac{2}{9} \div \frac{2}{5} &= \frac{2}{9} \cdot \frac{5}{2} \\ &= \frac{2^1}{9} \cdot \frac{5}{1\cancel{2}} \\ &= \frac{5}{9} \end{aligned}$$

Multiply by the reciprocal of the divisor.

Divide the common factors.

Simplify.

b. Find $\frac{x}{3} \div \frac{x}{4}$.

$$\begin{aligned} \frac{x}{3} \div \frac{x}{4} &= \frac{x}{3} \cdot \frac{4}{x} \\ &= \frac{x^1}{3} \cdot \frac{4}{1\cancel{x}} \\ &= \frac{4}{3} = 1\frac{1}{3} \end{aligned}$$

Quick Check

4. Find each quotient. Simplify if possible.

a. $-\frac{1}{4} \div \frac{1}{2}$

b. $\frac{5a}{8} \div \frac{2}{3}$

c. $\frac{3b}{7} \div \frac{6}{7}$

To divide mixed numbers, change the mixed numbers to improper fractions before multiplying by the reciprocal of the divisor.

EXAMPLE Dividing Mixed Numbers

Find $1\frac{3}{4} \div (-2\frac{5}{8})$.

$$\begin{aligned} 1\frac{3}{4} \div (-2\frac{5}{8}) &= \frac{7}{4} \div (-\frac{21}{8}) \\ &= \frac{7}{4} \cdot (-\frac{8}{21}) \\ &= \frac{1\cancel{7}}{1\cancel{4}} \cdot -\frac{2\cancel{8}}{3\cancel{21}} = -\frac{2}{3} \end{aligned}$$

Change to improper fractions.

Multiply by $-\frac{8}{21}$, the reciprocal of $-\frac{21}{8}$.

Divide the common factors. Simplify.

Quick Check

5. Find each quotient. Simplify if possible.

a. $1\frac{1}{3} \div \frac{5}{6}$

b. $-1\frac{3}{5} \div 1\frac{1}{5}$

c. $12\frac{1}{2} \div 1\frac{2}{3}$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Examples 1 and 2
(pages 252 and 253)



for
Help

Example 3
(page 253)

Find each product. Simplify if possible.

- | | | | |
|-------------------------------------|---------------------------------------|--|---|
| 1. $\frac{2}{3} \cdot \frac{1}{5}$ | 2. $-\frac{1}{2}(\frac{3}{8})$ | 3. $-\frac{4}{7} \cdot -\frac{3}{5}$ | 4. $(-\frac{2}{3})(\frac{11}{13})$ |
| 5. $(-\frac{7}{8})(-\frac{4}{5})$ | 6. $\frac{12y}{25} \cdot \frac{5}{6}$ | 7. $\frac{9}{10} \cdot \frac{15x}{3}$ | 8. $\frac{5}{9}(\frac{9}{10})$ |
| 9. $5\frac{7}{8} \cdot \frac{6}{7}$ | 10. $2\frac{3}{4} \cdot 1\frac{1}{5}$ | 11. $-1\frac{2}{5} \cdot 2\frac{2}{7}$ | 12. $-3\frac{2}{5} \cdot -1\frac{2}{3}$ |

13. **Homework** Jim spends $\frac{3}{4}$ of an hour on homework. His older sister Gina spends $1\frac{2}{3}$ times as much on her homework as Jim spends on his. How much time does Gina spend doing her homework?

Example 4
(page 254)

Find each quotient. Simplify if possible.

- | | | | |
|--------------------------------------|---------------------------------------|--------------------------------------|--|
| 14. $\frac{1}{2} \div \frac{1}{3}$ | 15. $\frac{5}{8} \div \frac{3}{4}$ | 16. $-\frac{3}{4} \div \frac{1}{3}$ | 17. $\frac{11}{12} \div (-\frac{7}{8})$ |
| 18. $\frac{3}{4} \div \frac{8}{9}$ | 19. $\frac{3}{4} \div \frac{1}{2}$ | 20. $\frac{2t}{5} \div \frac{2}{5}$ | 21. $\frac{1}{x} \div \frac{3}{x}$ |
| 22. $12\frac{2}{3} \div \frac{3}{4}$ | 23. $1\frac{3}{8} \div 2\frac{1}{16}$ | 24. $-1\frac{7}{9} \div \frac{8}{9}$ | 25. $-3\frac{2}{3} \div (-2\frac{4}{9})$ |
| 26. $3\frac{1}{2} \div \frac{4}{21}$ | 27. $7\frac{2}{3} \div 1\frac{5}{6}$ | 28. $6\frac{3}{4} \div \frac{9}{10}$ | 29. $1\frac{4}{5} \div (-1\frac{1}{2})$ |

Example 5
(page 254)

B Apply Your Skills

Find each product. Simplify if possible.

- | | | | |
|--------------------------------------|---------------------------------------|--|--|
| 30. $\frac{6x}{7} \cdot \frac{1}{3}$ | 31. $-\frac{2}{3} \cdot \frac{9}{10}$ | 32. $\frac{8}{9} \cdot \frac{15}{28}$ | 33. $-1\frac{1}{4} \cdot 6\frac{2}{3}$ |
| 34. $\frac{4}{t} \cdot \frac{3t}{8}$ | 35. $\frac{4a}{9} \cdot \frac{3}{10}$ | 36. $1\frac{3}{5} \cdot (-2\frac{1}{2})$ | 37. $(-\frac{7}{12})(-\frac{5}{6})$ |

38. **Number Sense** One granola bar weighs $1\frac{1}{2}$ oz. What is the weight of six granola bars?

Find each quotient. Simplify if possible.

- | | | | |
|--------------------------------------|--|-------------------------------------|--------------------------------------|
| 39. $-\frac{1}{2} \div \frac{2}{3}$ | 40. $\frac{10}{13} \div \frac{15}{26}$ | 41. $-\frac{5}{6} \div \frac{4}{9}$ | 42. $\frac{4}{9x} \div \frac{2}{3x}$ |
| 43. $\frac{2}{5} \div \frac{15}{16}$ | 44. $-\frac{6n}{7} \div \frac{n}{3}$ | 45. $\frac{2}{9} \div \frac{w}{3}$ | 46. $\frac{3}{8} \div \frac{6}{32}$ |

Mental Math Simplify each expression.

- | | | | |
|-------------------------------------|------------------------------------|---------------------------------------|--------------------------------------|
| 47. $\frac{1}{2} \cdot \frac{2}{5}$ | 48. $\frac{1}{2} \div \frac{2}{5}$ | 49. $10 \cdot \frac{1}{4}$ | 50. $10 \div \frac{1}{4}$ |
| 51. $\frac{5}{8} \cdot \frac{3}{5}$ | 52. $\frac{5}{8} \div \frac{3}{5}$ | 53. $\frac{3}{7} \cdot \frac{12}{21}$ | 54. $\frac{3}{7} \div \frac{12}{21}$ |

55. **Construction** A cable television crew has to install cable along a road $1\frac{1}{2}$ mi long. The crew takes a day to install each $\frac{1}{4}$ mi of cable. How many days will the installation take?

- (A) 6 days (B) $1\frac{3}{4}$ days (C) $\frac{3}{4}$ day (D) $\frac{3}{8}$ day

56. a. Write an expression for the following: The product of $\frac{1}{2}a$ and 3 is decreased by the quotient $a \div (-4)$.
b. Evaluate your expression for $a = 3$.

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57. A cheetah can run as fast as 64 mi/h. At that speed, how far could a cheetah run in $\frac{1}{16}$ h? $\frac{1}{30}$ h?

58. You are hiking along a trail that is $13\frac{1}{2}$ mi long. You plan to rest every $2\frac{1}{4}$ mi. How many rest stops will you make?

C Challenge

59. **Writing in Math** Why must you change mixed numbers to improper fractions before multiplying or dividing them?

60. a. **Patterns** Find each quotient: $\frac{1}{2} \div 2$, $\frac{1}{2} \div 3$, $\frac{1}{2} \div 4$, and $\frac{1}{2} \div 5$.
b. Explain what happens to the quotients as the divisor increases in value.

61. **Reasoning** Write a multiplication equation and a division equation that you could use to show the result of cutting four melons into eight equal slices each.

62. **Open-Ended** Find two fractions greater than $\frac{1}{2}$ with a product less than $\frac{1}{2}$.



Test Prep

Multiple Choice

63. Which quotient does NOT equal 1?

A. $2\frac{3}{4} \div \frac{11}{4}$

B. $\frac{3}{8} \div 0.375$

C. $\frac{7}{8} \div \frac{7}{8}$

D. $-1\frac{2}{3} \div (-\frac{3}{5})$

64. Which expression simplifies to $\frac{x}{3}$?

F. $\frac{5x}{36}(\frac{5}{12})$

G. $\frac{x}{6}(2\frac{2}{5})$

H. $\frac{5x}{36}(2\frac{2}{5})$

J. $\frac{36}{5x}(2\frac{2}{5})$

Short Response

65. a. A family wants to travel 300 miles. They drive at an average speed of 65 mi/h for $3\frac{1}{2}$ hours. Have they driven far enough?
b. Explain your answer for part (a).

66. Natasha's bedroom floor is $10\frac{1}{2}$ ft by $14\frac{3}{4}$ ft. She buys 160 ft² of carpet. Does she have enough carpet to cover the floor? Explain.

Mixed Review



Lesson 5-3

Add or subtract.

67. $\frac{4}{5} + \frac{6}{7}$

68. $\frac{10}{13} - \frac{25}{26}$

69. $-\frac{3}{10} + \frac{3}{5}$

70. $\frac{16}{21} - \frac{5}{7}$

Lesson 2-6

Simplify each fraction.

71. $\frac{10}{12}$

72. $\frac{24}{40}$

73. $\frac{45}{10}$

74. $\frac{12}{50}$

75. $\frac{34}{51}$

76. $\frac{105}{135}$

Lesson 2-7

77. Hal's age is three times Ida's age. In 8 years Hal will be twice as old as Ida. How old is Hal?

78. **Personal Finance** You spent $\frac{1}{4}$ of your money on lunch. After lunch, you gave half of what you had left to a friend, and then you spent \$3 on a book. You have \$4.50 left. How much money did you have before lunch?

